Structural Analysis

Lecture Series

SA66: The Analysis of a Frame

Under Snow Load

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http://www.Lab101.Space

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2020

Structural Analysis – SA66
Frame Analysis – Snow Load
Prerequisite: SA45 through SA50

In this lecture, we will discuss the analysis of a frame structure subject to snow load. Snow can exert a substantial load on building roofs, producing additional internal forces in the skeleton of the structure.

Most design standards offer provisions for calculating the roof snow load, as well as identifying the critical load patterns that must be considered when designing a structure. Such standards allow us to determine the required load-carrying capacity of the structural members so that they can be designed properly.

Figure 1: A house with a flat roof carrying snow load

Herein, we present a method for calculating the roof snow load and examine the load patterns that govern the design of the structural skeleton of the house shown in Figure 1.

We then select and analyze one of the skeleton's interior frames to determine the critical bending moments that develop in the beams and columns under snow load.

Figure 2: Four steel frames forming the load-carrying skeleton of a house

For structural analysis, we use the matrix displacement method. Here, however, we only present the results of the analysis since we have discussed the details of this method in previous lectures. You can learn about the specifics of the matrix displacement method by reviewing Lectures SA45 through SA50.

Case Illustration

Consider a house located in the Northeast region of the United States where snow could deposit a significant amount of load on the roof of the structure. As shown in Figure 2, the skeleton of the house consists of four parallel frames. Let us analyze one of the interior frames under snow load.

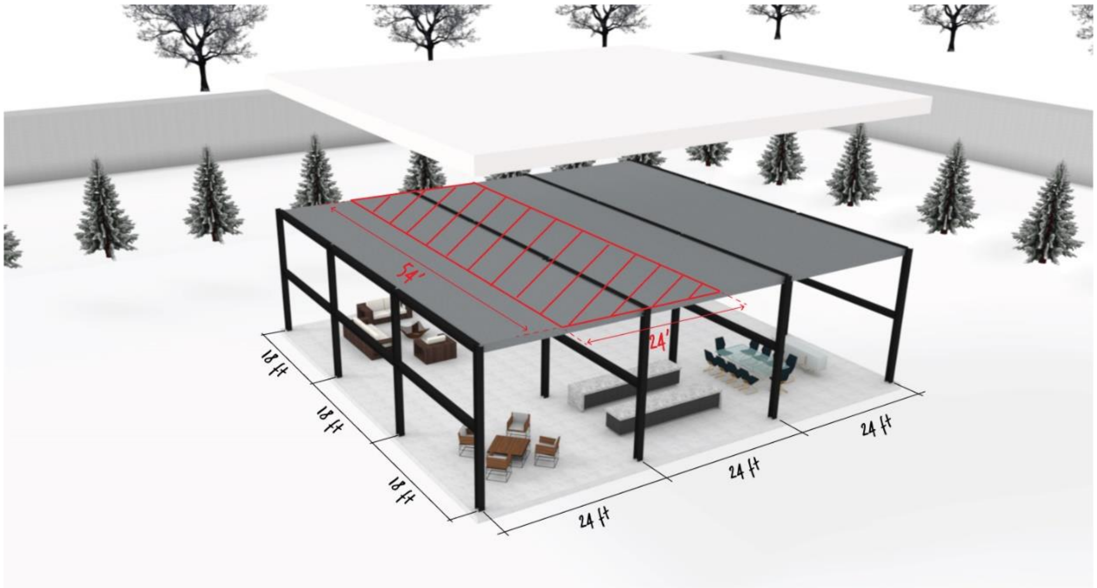

Figure 3: The roof tributary area for an interior frame of a house

As depicted in Figure 3, the roof snow load is transferred to the supporting frames via a flat rectangular concrete slab. Consequently, each interior frame is responsible for carrying the load of a 54 feet by 24 feet rectangular area. The target interior frame and its respective snow load are depicted in Figure 4.

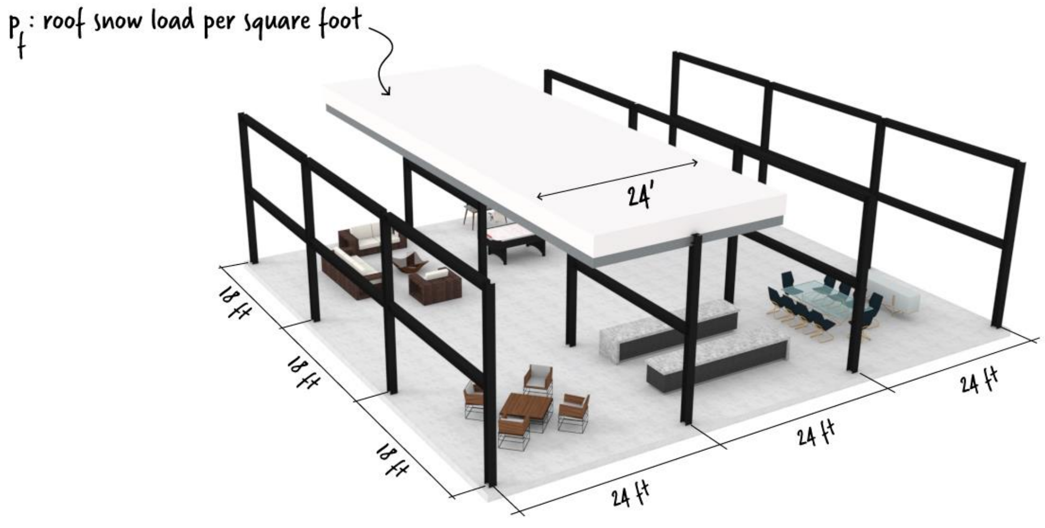

Figure 4: An interior frame of a house carrying a portion of the roof snow load

If the weight per square foot of snow is denoted as p_f, the resulting uniformly distributed line load on the frame can be written as $24p_f$. This is the magnitude of the uniformly distributed snow load that will be used to analyze this frame (see Figure 5). We assume that the frame is fixed at its base.

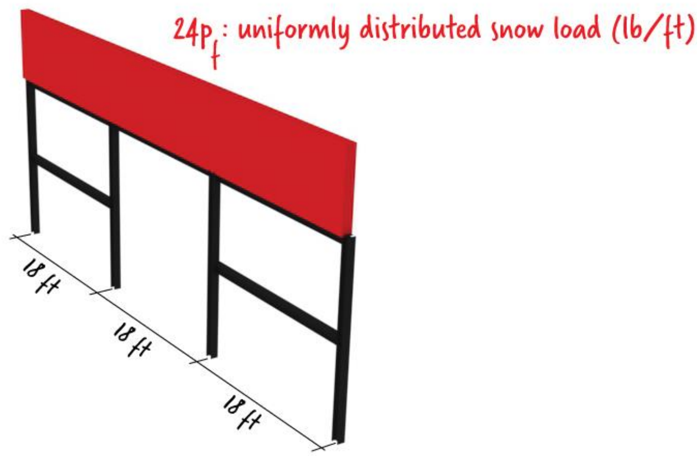

Figure 5: A building frame subject to a uniformly distributed snow load

To determine p_f, we are going to use a design standard published by the American Society of Civil Engineers referred to as ASCE 7-16. Here, roof snow load can be expressed in terms of ground snow load using the following equation:

$$p_f = 0.7 C_e C_t I_s p_g$$ ASCE 7-16, Equation 7.3-1

The above equation is a product of four factors:

C_e refers to the exposure factor, and it is a measure of the nature of the terrain, location, and exposure of the structure. For a detached single-family house located in a suburban area as used in this example, the exposure factor is 1 (see ASCE 7-16, Table 7.3-1).

C_t indicates the thermal factor, and it is a function of the ambient temperature of the building. Snow tends to melt and accumulate less on the roof of a heated home than on a building that has a lower ambient temperature. For a residential building, a thermal factor of 1.0 can be used (see ASCE 7-16, Table 7.3-2).

I_s is the importance factor, and it is a measure of the risk to human life in the event of structural failure. For a typical residential house, the importance factor is 1 (see ASCE 7-16, Tables 1.5-1, and 1.5-2).

p_g refers to the ground snow load, and its value can be determined using a map given in the standard. Since the house in our example is located in a suburban area near New York City, a ground snow load of 30 lb/ft^2 should be used.

By substituting the above values in Equation 7.3-1, we calculate the roof snow load to be 21 lb/ft^2.

For flat or low-sloped roofs, we need to ensure that the design snow load exceeds the minimum required load. ASCE 7-16 provides the following equation for calculating the minimum snow load.

$$p_m = \begin{cases} I_s p_g & \text{if } p_g \leq 20 \text{ lb/ft}^2 \\ 20(I_s) & p_g > 20 \text{ lb/ft}^2 \end{cases}$$ ASCE 7-16, Provision 7.3.4

Since p_g is greater than 20, the second equation above is applicable. Therefore, the minimum snow load (p_m) is: 20(1) = 20 lb/ft^2. This load is less than the calculated p_f; therefore, the computed roof snow load (21 lb/ft^2) will govern the design.

Figure 6 shows a 2D view of the frame subjected to a uniformly distributed snow load of (24 ft)(21 lb/ft^2) = 504 lb/ft.

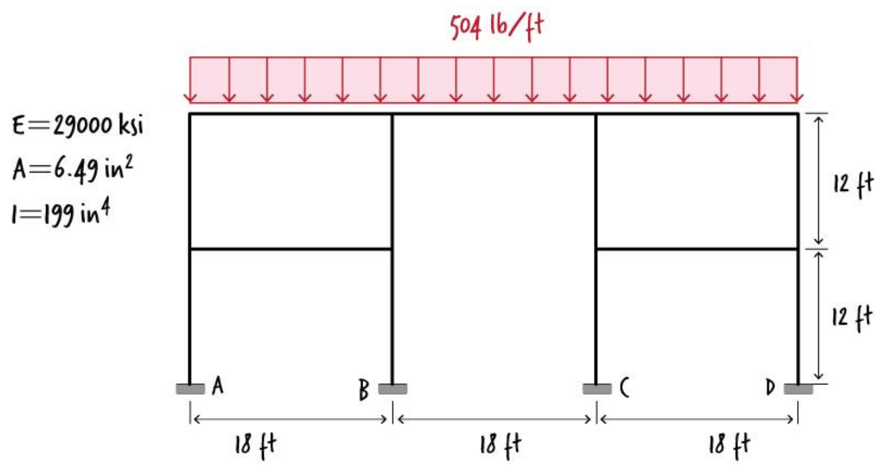

Figure 6: A steel frame under a uniformly distributed load (Load Case 0)

The frame uses structural steel members, each having a cross-sectional area of 6.49 in^2 and a moment of inertia of 199 in^4. The steel modulus of elasticity is assumed to be 29000 ksi.

Clearly, we must analyze the frame under the loading case shown in Figure 6. The case assumes that snow has covered the entire tributary roof area for the frame. We will refer to this as Load Case 0.

In addition, let's contemplate a scenario in which snow only partially covers the roof. To understand why partial loading must be anticipated, imagine a roof beam with an overhang on either side such as shown in Figure 7.

Suppose the roof is covered with snow, exerting a uniformly distributed load of 100 lb/ft on the beam. If we draw the beam's moment diagram, we find that the maximum bending moment, 3250 lb-ft, is at the midpoint of the span, as shown in Figure 7.

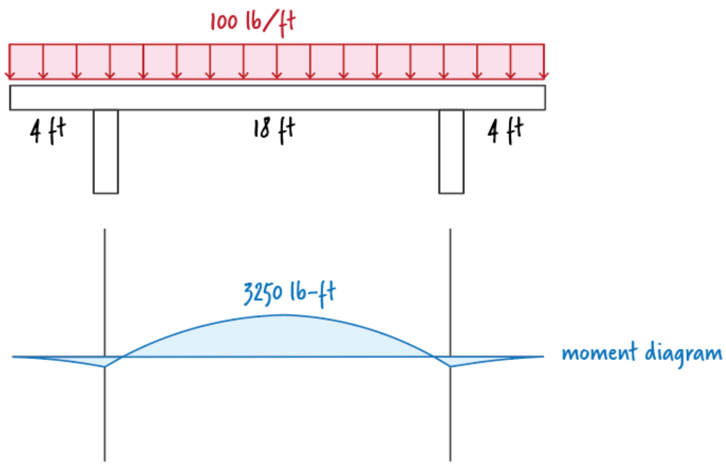

Figure 7: A roof beam with overhangs under full snow load

Now, consider what would occur if the snow is removed from the two overhang areas, but the rest of the roof is left untouched (see Figure 8). This would cause the maximum moment in the beam to increase from 3250 to 4050 lb-ft.

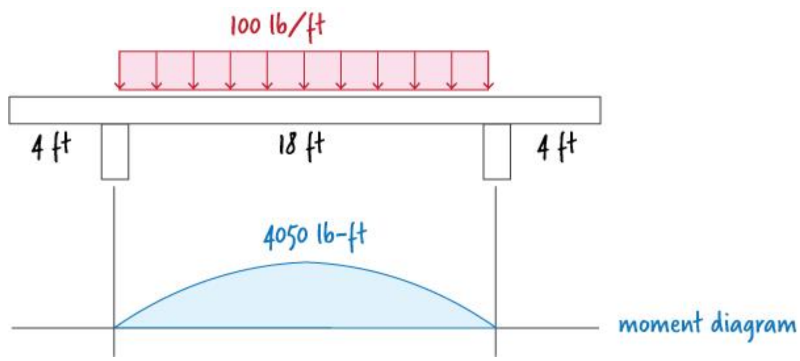

Figure 8: A roof beam with overhangs under partial snow load

The above example illustrates how the internal member forces can increase when part of the snow is removed from the roof.

Per ASCE 7-16, three partial loading cases should be examined for the house under consideration, in addition to the full loading case shown in Figure 6.

Case 1: Full snow load is present on an exterior span while all other spans carry half the snow load. This case results in two loading scenarios. If we take beam (9-10) as the exterior span, this creates the loading scenario shown in Figure 9a. If, however, we choose beam (11-12) as the exterior span, we arrive at the loading scenario shown in Figure 9b.

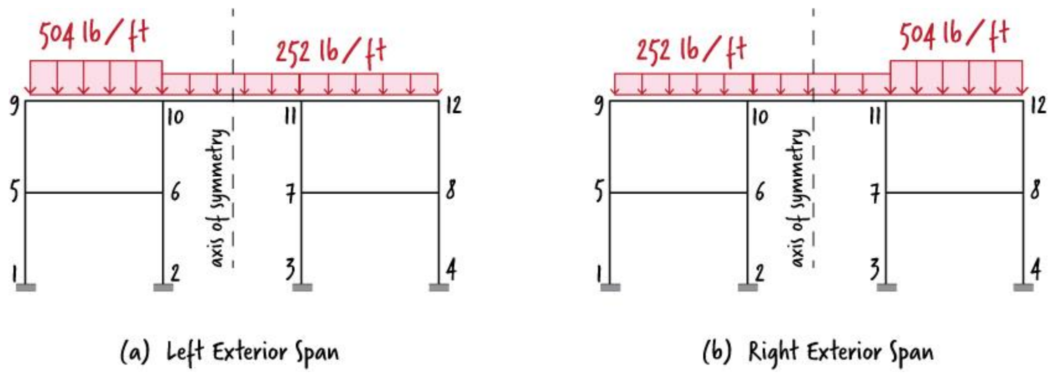

Figure 9: Partial snow loading for a frame (Case 1)

Since the frame is symmetrical about the vertical line (axis of symmetry) that passes through the midpoint of member (10-11), we only need to analyze the frame under one of the two loading scenarios. For example, under the loading scenario shown in Figure 9a, the analysis of the frame reveals that the maximum positive moment in member (6-10) is 5 k-ft. This would also be the value for the maximum positive moment in member (7-11) under the second loading scenario, being that member (7-11) is the mirror image of member (6-10) about the frame's axis of symmetry.

For the purposes of analysis, we will refer to the scenario shown in Figure 9a as Load Case 1.

Case 2: Half of the snow load is placed on an exterior span while all other spans carry the full snow load. This case is similar to case 1 in that we can anticipate two loading scenarios. However, we only need to perform the necessary computations for one of them.

The two loading scenarios are shown in Figure 10. We will be performing the analysis computations on loading scenario (a).

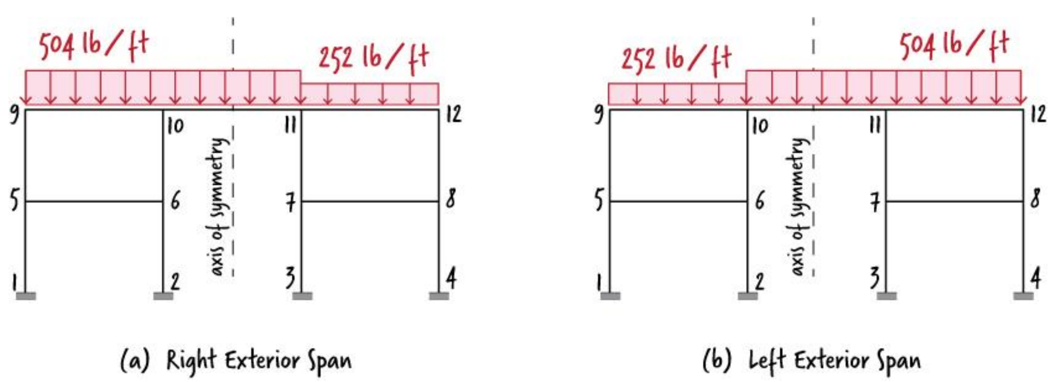

Figure 10: Partial snow loading for a frame (Case 2)

Case 3: Any two adjacent spans are subjected to the full snow load while the remaining spans carry only half the load. For our frame, this loading case turns out to be identical to Case 2 (see figure above). Therefore, we can omit it from further consideration.

ASCE 7-16 offers provisions for additional loading cases for sloped roofs. However, since the roof in our example is flat, those cases are not applicable here. Therefore, the frame must be analyzed under three distinct loading cases shown in Figures 6, 9a, and 10a.

For the frame analysis, we use the matrix displacement method. The details of the method were presented in previous lectures (see SA45 through SA50) so it won't be reiterated. To facilitate the calculations, we use iFrame, an application program developed for educational purposes that is used to analyze the frame for each loading case. You can learn more about the program by watching video IT02.

Analysis Results

For Load Case 0, the moment diagram for each beam and column is shown in Figure 11. For better visualization, the column diagrams are shown in blue, and the beam diagrams are drawn in red. Except for the three beams along the top of the frame where the distributed load resides, the moment diagram has a linear shape in each member. The moment diagram for each beam along the top of the frame has a quadratic shape, with negative values at the ends of the member and a maximum positive value somewhere near the mid-span.

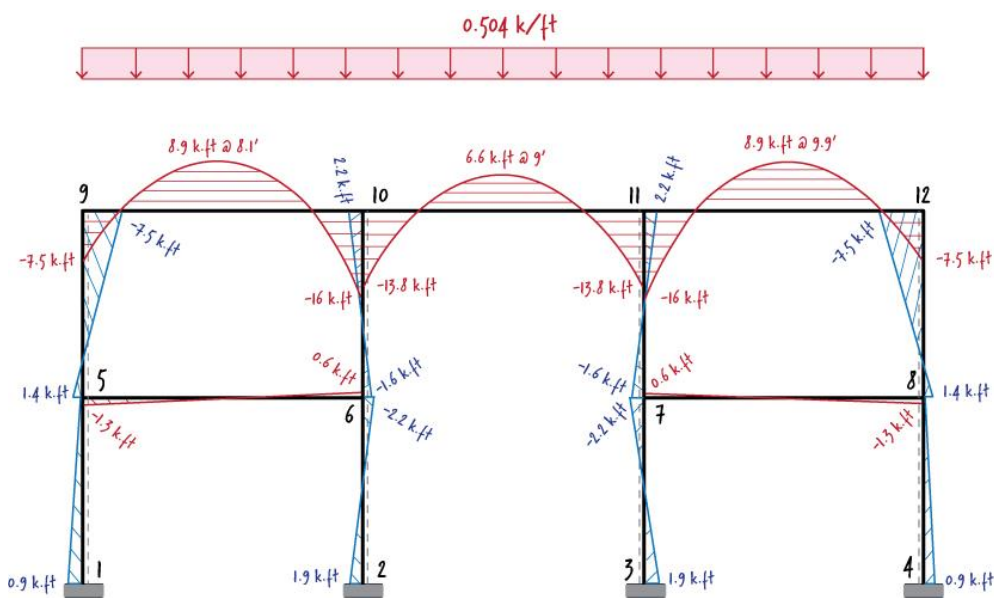

Figure 11: The frame moment diagram for Load Case 0

The moment diagrams for the other two loading cases follow the same overall pattern; however, in each case, the moment values vary for each member.

For example, note the maximum positive and negative moments for member (9-10), which are +8.9 k-ft and -16 k-ft, respectively. Now, compare these values with the values for the same member for Load Case 1, shown in Figure 12. Note that the maximum moment in the member has gone up to 9.4 k-ft, while the negative moment has increased to -14.1 k-ft.

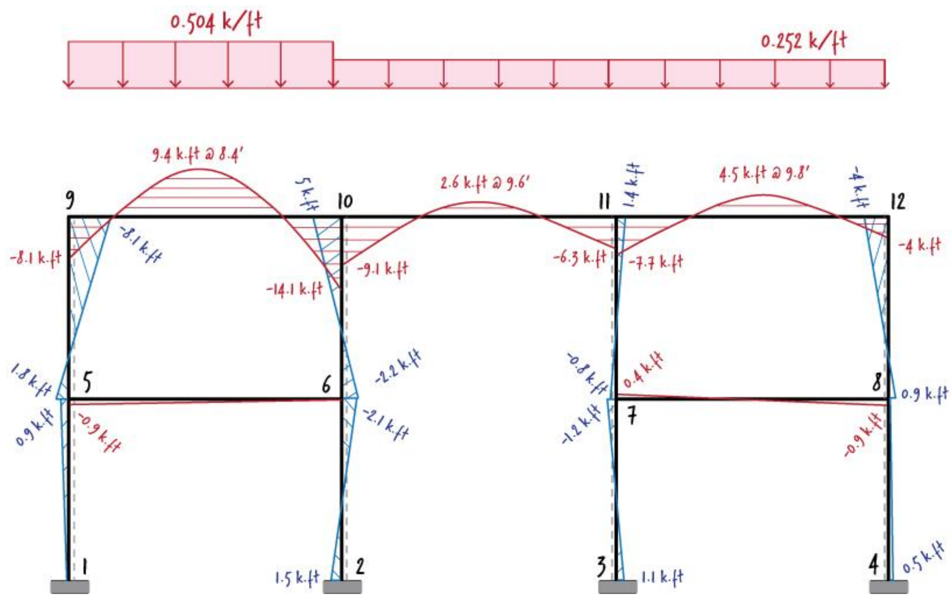

Figure 12: The frame moment diagram for Load Case 1

Under Load Case 2 (see Figure 13), the positive moment for the same member drops to 8.8 k-ft, while the negative moment reaches the member's most critical value: -16.5 k-ft. As you can see, the moment values in a typical member fluctuate as the snow pattern on the roof changes.

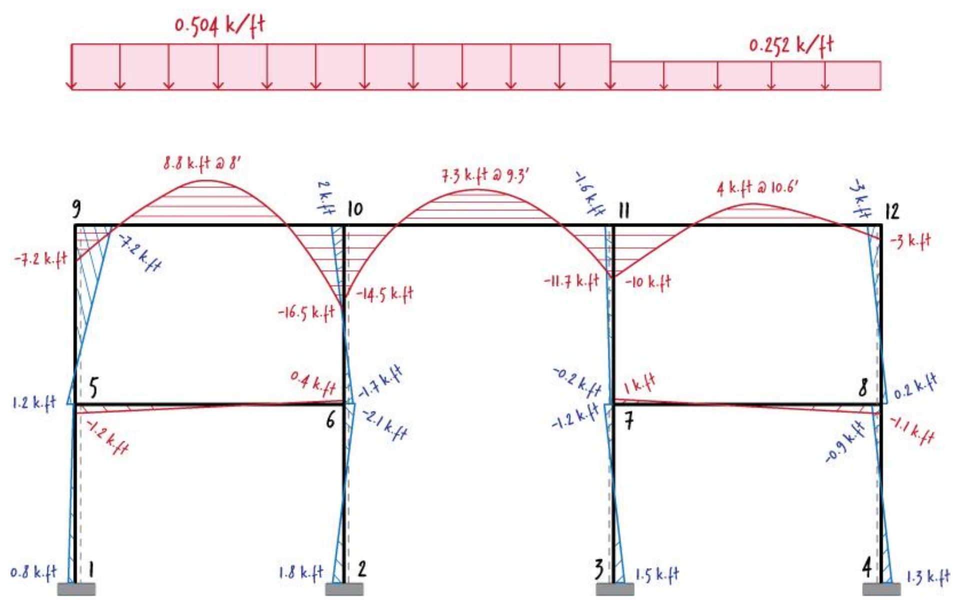

Figure 13: The frame moment diagram for Load Case 2

A comparison of the three moment diagrams reveals the following maximum positive and negative moments for the left half of the frame (see Figure 14).

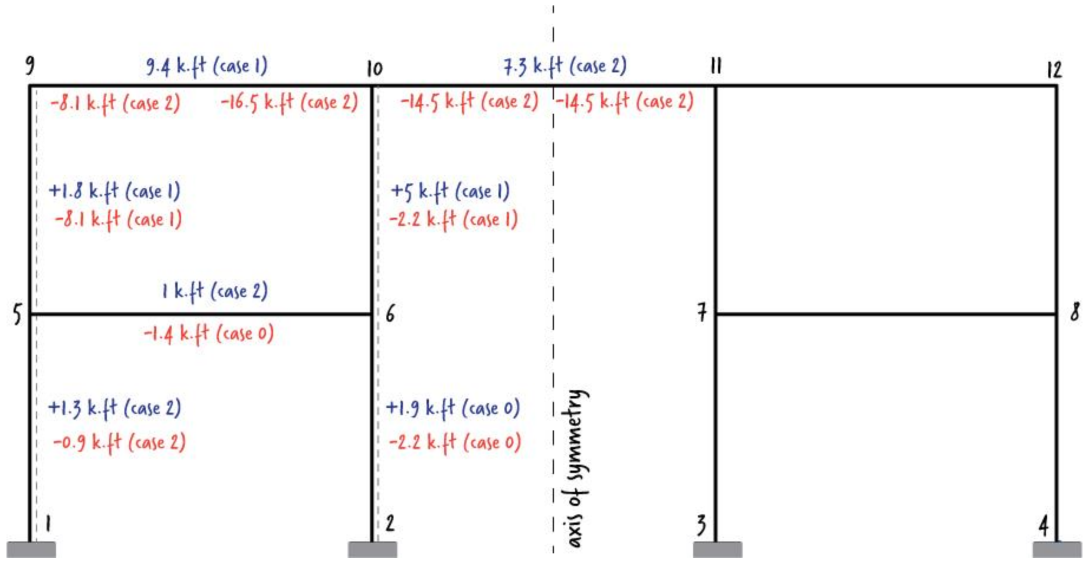

Figure 14: Critical moment values for the left half of a symmetrical frame

Since the frame is symmetrical, we can use the same set of values for the right half of the frame. The critical moment values for the entire frame are shown in Figure 15. The diagram shows the maximum positive and negative moments for each beam and column when the frame is subjected to snow load only.

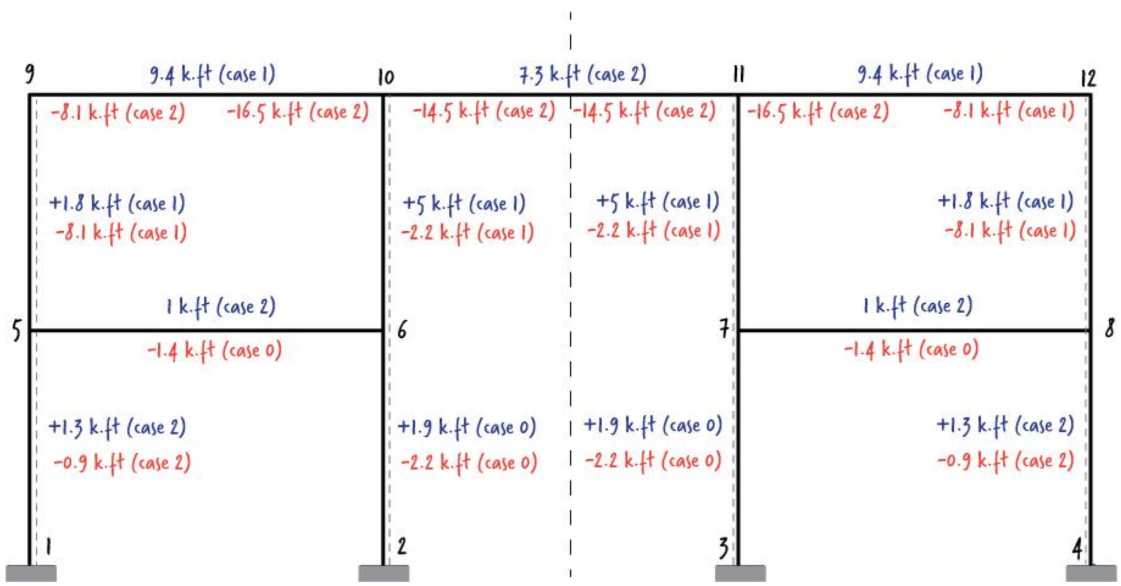

Figure 15: Critical moment values for an interior frame of house under wind load

Understandably, if we were to actually design the house, we would consider other load types and load combinations to ensure the safety of the system under dead load, live load, wind load, and other relevant loads governing the design of the structure.