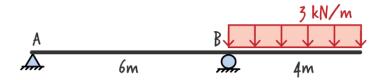
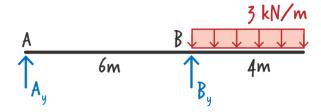
Statics—ST10 (Solution for Exercise A) Shear and Moment Equations for Statically Determinate Beams

Write the shear and moment equations for the statically determinate beam shown below.



Solution

Draw the beam's free-body diagram.



The beam has two non-zero unknown reaction forces. Write the necessary equilibrium equations for determining the support reactions.

$$\sum F_{y} = A_{y} + B_{y} - 4(3) = 0$$

$$\sum M_{\partial B} = (3)(4)(4/2) + 6A_{y} = 0$$

Solving the above equations for the reaction forces, we get the following:

$$A_y = -4 \text{ kN}$$
 $B_y = 16 \text{ kN}$

We need two sets of shear and moment equations - one for the left segment and one for the right segment of the beam. For the left segment, the free-body diagram can be drawn as shown below:

$$\begin{array}{cccc}
A & & & & & \\
\downarrow & & & & & \\
4 & kN & & & & \\
\end{array}$$

The equilibrium equations for the above free-body diagram are as follows:

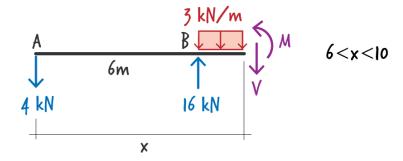
$$\sum_{M_{2A}=(x)(V)-M=0}$$

Solving these equations for V and M, we get the shear and moment equations for the left segment of the beam in terms of x where x is between 0 and 6.

$$V(x)=-4 \text{ kN}$$

 $M(x)=-4x \text{ kN.m}$

For the right segment of the beam, we cut the segment at an arbitrary point and draw the needed free-body diagram as shown below:



The equilibrium equations for the above diagram can be written as follows:

$$\sum F_{y} = -4 + 16 - 3(x - 6) - V = 0$$

$$\sum M_{ab} = (3)(x - 6) \frac{x - 6}{2} + (x - 6)(V) - M - 6(4) = 0$$

Solving these equations for shear and moment, we get the following:

$$V(x)=30-3x \text{ kN}$$

$$M(x)=-\frac{3}{2}(x-10)^2 \text{ kN.m}$$

Here is the summary of the results:

$$V(x) = \begin{cases} -4 & kN & 0 < x < 6 \\ 30 - 3x & kN & 6 < x < 10 \end{cases}$$

$$V(x) = \begin{cases} -4 & kN & 0 < x < 6 \\ 30 - 3x & kN & 6 < x < 10 \end{cases}$$

$$M(x) = \begin{cases} -4x & kN.m & 0 \le x \le 6 \\ -\frac{3}{2}(x - 10)^2 & kN.m & 6 \le x \le 10 \end{cases}$$