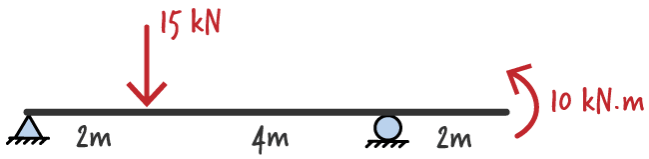
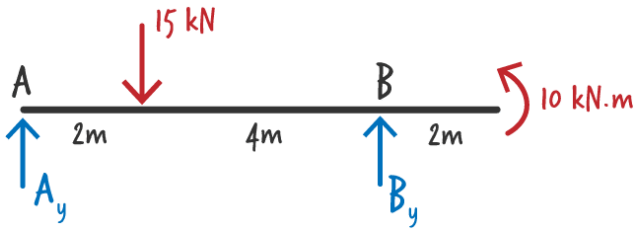
## Statics– ST10 (Solution for Exercise C)
### Shear and Moment Equations for Statically Determinate Beams

Write the shear and moment equations for the statically determinate beam shown below.




## Solution

Draw the beam's free-body diagram.



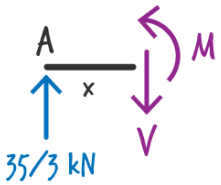

The beam has two non-zero unknown reaction forces. Write the necessary equilibrium equations for determining the support reactions.

$$\sum F_y = A_y + B_y - 15 = 0$$
$$\sum M_{@A} = (15)(2) - 10 - 6B_y = 0$$

Solving the above equations for the reaction forces, we get the following:

$$A_y = 35/3 \text{ kN}$$
$$B_y = 10/3 \text{ kN}$$

We need three sets of shear and moment equations: one for the left segment, one for the middle segment, and one for the right segment of the beam. For the left segment, the free-body diagram can be drawn as shown below:




$0 < x < 2$

The equilibrium equations for the above free-body diagram are as follows:

$$\sum F_y = \frac{35}{3} - V = 0$$

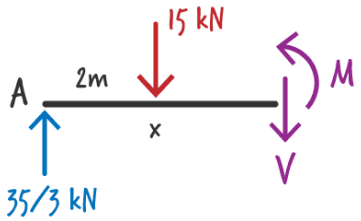
$$\sum M_{@A} = (x)(V) - M = 0$$

Solving these equations for $V$ and $M$, we get the shear and moment equations for the left segment of the beam in terms of $x$ where $x$ is between 0 and 2.

$$V(x) = \frac{35}{3} \text{ kN}$$

$$M(x) = \frac{35}{3}x \text{ kN.m}$$

For the middle segment of the beam, we cut the segment at an arbitrary point and draw the needed free-body diagram as shown below:




$2 < x < 6$

The equilibrium equations for the above diagram can be written as follows:

$$\sum F_y = \frac{35}{3} - 15 - V = 0$$

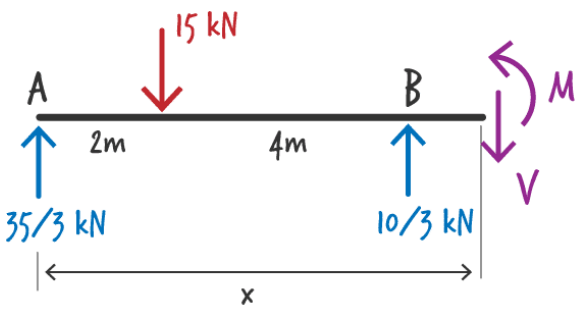
$$\sum M_{@A} = (2)(15) + (x)(V) - M = 0$$

Solving these equations for shear and moment, we get the following two equations:

$$V(x) = -\frac{10}{3} \text{ kN}$$

$$M(x) = 30 - \frac{10}{3}x \text{ kN.m}$$

For the right segment of the beam, the free-body diagram can be drawn as shown below:




$$6<x<8$$

The equilibrium equations for the above diagram can be written as follows:

$$\sum F_y = \frac{35}{3} - 15 + \frac{10}{3} - V = 0$$

$$\sum M_{@A} = (2)(15) - (6)\left(\frac{10}{3}\right) + (x)(V) - M = 0$$

Solving these equations for shear and moment, we get the following two equations:

$$V(x) = 0$$
$$M(x) = 10 \text{ kN.m}$$

Here is the summary of the results:

$$V(x) = \begin{cases} \frac{35}{3} \text{ kN} & 0<x<2 \\ -\frac{10}{3} \text{ kN} & 2<x<6 \\ 0 & 6<x<8 \end{cases} \qquad M(x) = \begin{cases} \frac{35}{3}x \text{ kN.m} & 0 \le x \le 2 \\ 30 - \frac{10}{3}x \text{ kN.m} & 2 \le x \le 6 \\ 10 \text{ kN.m} & 6 \le x \le 8 \end{cases}$$