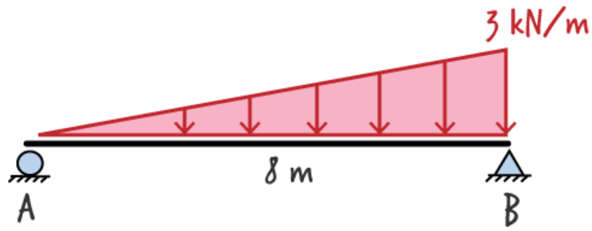
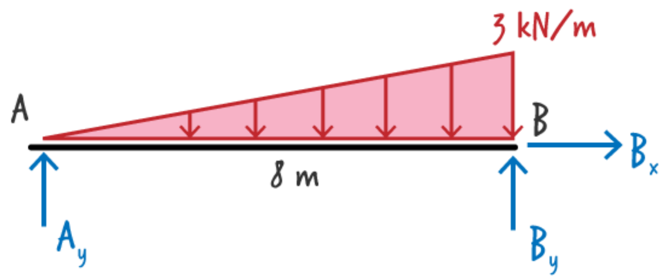
Statics– ST11 (Solution for Exercise Problem 1)
Shear Diagram for Statically Determinate Beams

Draw the shear diagram for the statically determinate beam shown below.

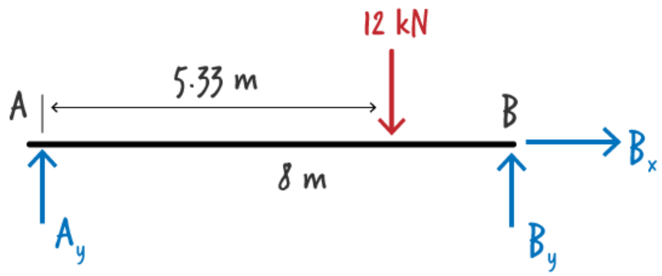


Solution

Draw the beam's free-body diagram.



Replace the distributed load with its equivalent concentrated load. The magnitude of the concentrated load is equal to the area of the triangle; the location of the load is the geometric center of the triangle.



Now, write and solve the static equilibrium equations for the unknown support reactions.

$$\sum F_x = B_x = 0$$

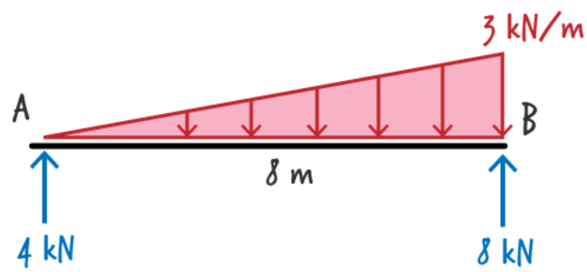
$$\sum F_y = A_y + B_y - 12 = 0$$

$$\sum M_{@A} = (12)(5.33) - 8B_y = 0$$

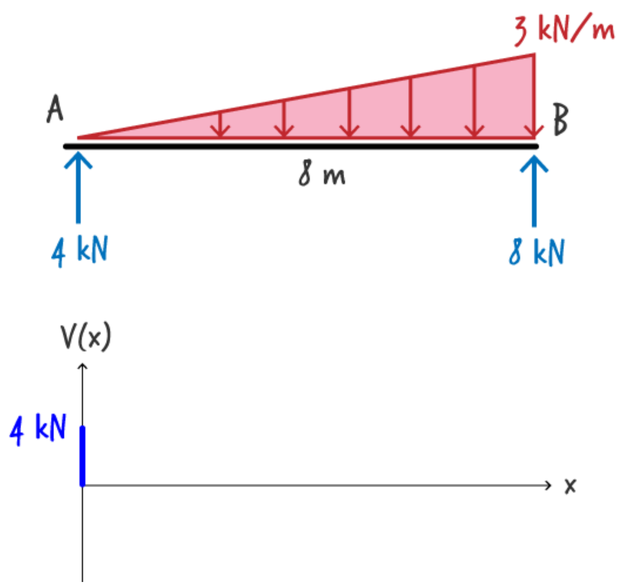
Solving the above equations for the reaction forces, we get:

$$B_x = 0$$
$$A_y = 4 \text{ kN}$$
$$B_y = 8 \text{ kN}$$

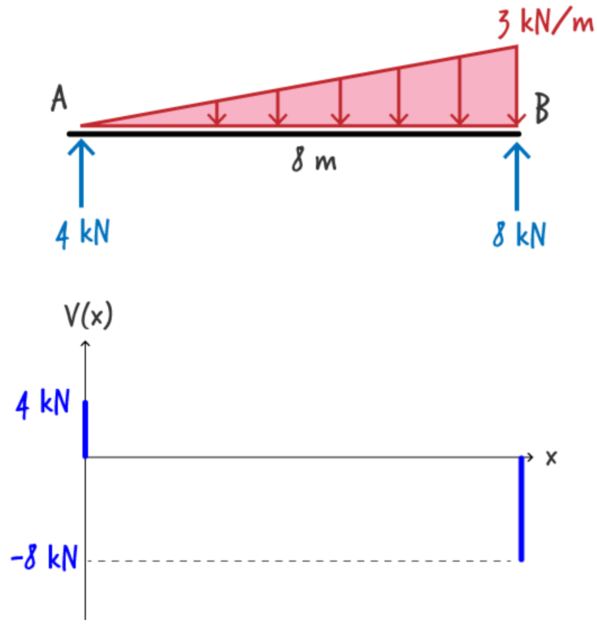
Knowing the support reactions, now draw the complete free-body diagram for the beam using the distributed load.



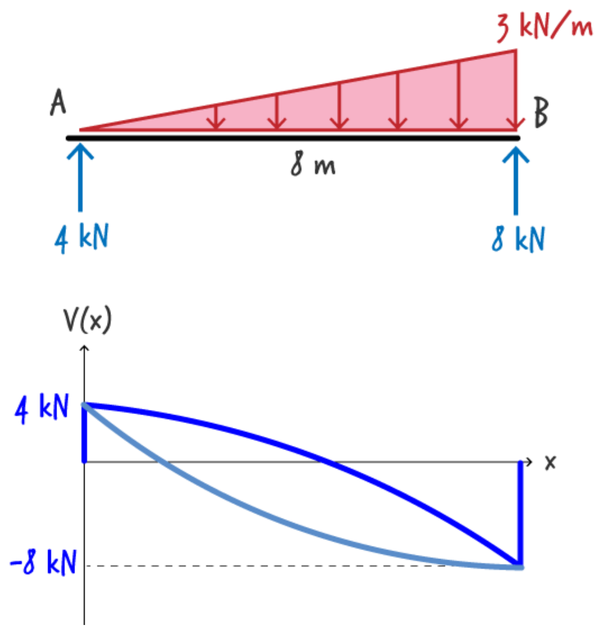
To draw the shear diagram, we start from the left end of the beam. Shear at the left end of AB is positive 4 kN, since there is an upward force of 4 kN at A.



Shear at the right end of the beam equals shear at the left end minus the area of the triangle representing the load. The area is (3 kN/m)(8 m)/2 = 12 kN. So, shear at the right end of AB equals 4 – 12 = -8 kN.



Since the load varies linearly, the shear diagram is going to have a linearly varying slope. This means, the diagram itself takes the shape of a quadratic equation (a second-degree polynomial) which has a linearly varying slope.



Note that the quadratic curve can be drawn in two ways. In the above diagram, the curve shown in light blue has an increasing slope. At A the curve has a large negative slope which increases toward zero as we approach Point B. The curve shown in dark blue has a decreasing slope. Its slope is close to zero at A, and becomes more negative as we approach B.

To determine which curve correctly represents the shear diagram, we need to examine the free-body diagram. If the distributed load is increasing in value as we move from A to B, then the curve with an increasing slope is the correct shape. If the load is decreasing in value from A to B, then the correct curve is the one with decreasing slope.

The triangular load, since it is downward (negative), has a decreasing magnitude as we move A to B. Therefore, the shear diagram takes the shape of the dark blue curve, the one with a decreasing slope, as shown below.