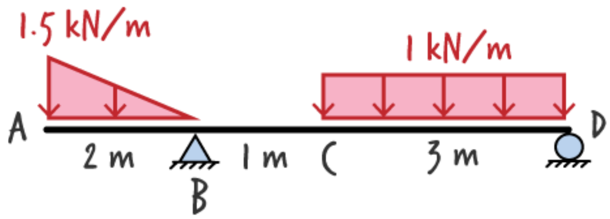
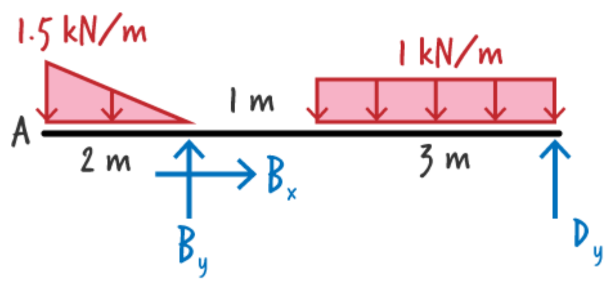
Statics– ST11 (Solution for Exercise Problem 2)
Shear Diagram for Statically Determinate Beams

Draw the shear diagram for the statically determinate beam shown below.

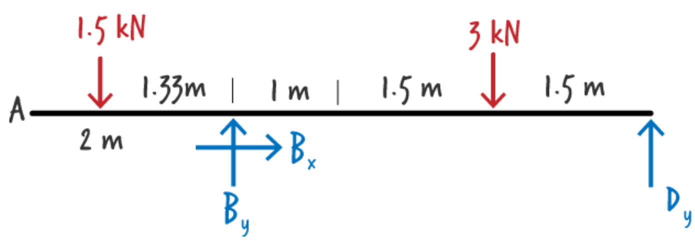


Solution

Draw the beam’s free-body diagram.



Replace the distributed load with its equivalent concentrated load. The magnitude of the concentrated load equals the area of the triangle representing the load, and the location of the load is the geometric center of the triangle.



Now, write and solve the static equilibrium equations for the unknown support reactions.

$$\sum F_x = B_x = 0$$

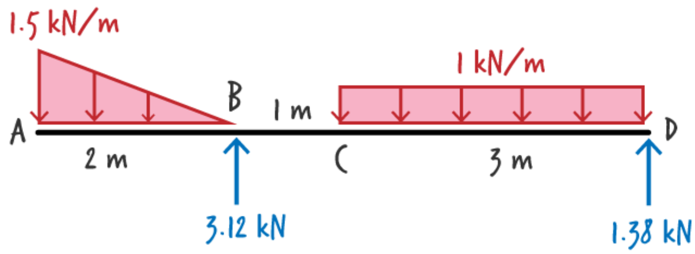
$$\sum F_y = B_y + D_y - 3 - 1.5 = 0$$

$$\sum M_{@D} = (1.5)(5.33) - 4B_y + 3(1.5) = 0$$

Solving the above equations for the reaction forces, we get:

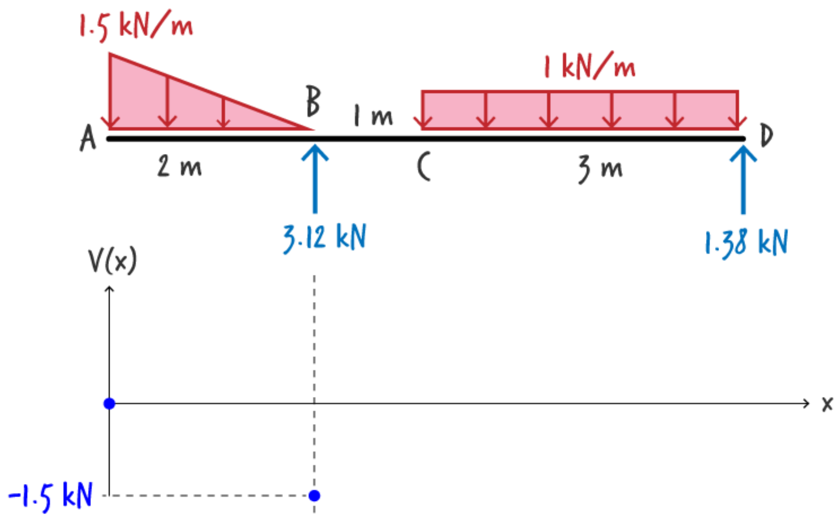
$$A_x = 0$$
$$B_y = 3.12 \text{ kN}$$
$$D_y = 1.38 \text{ kN}$$

Knowing the support reactions, now draw the complete free-body diagram for the beam using the distributed loads.



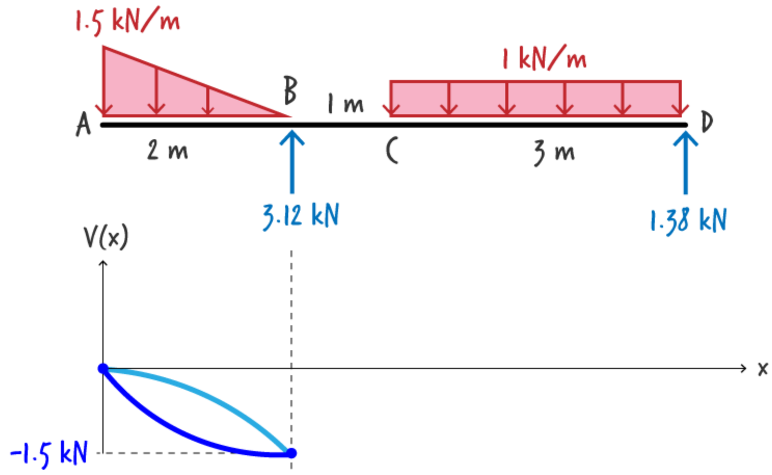
To draw the shear diagram, we start from the left end of the beam. Shear at the left end of AB is zero, since there is an upward concentrated force at A.

Shear at the right end of AB is equal to the shear at the left end minus the area of the triangle representing the load. The area is (1.5 kN/m)(2 m)/2 = 1.5 kN. So, shear at the right end of AB equals 0 – 1.5 = -1.5 kN. The end shear forces for AB are shown in figure below.



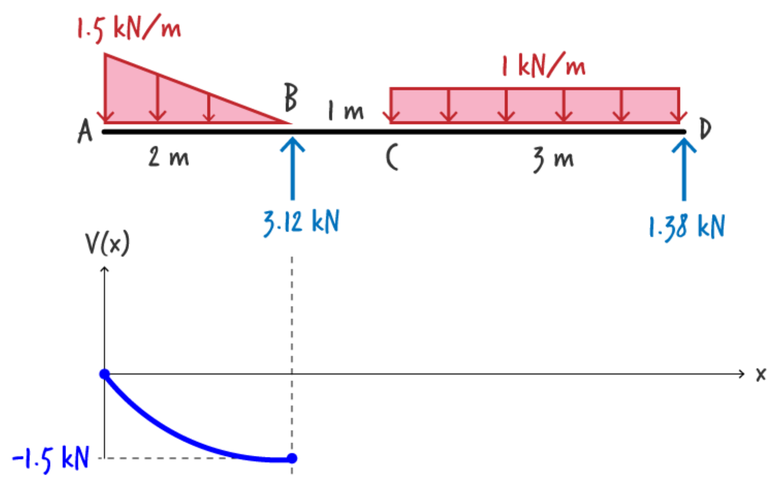
Since the load varies linearly in segment AB, the shear diagram is going to have a linearly varying slope in that segment. This means, the diagram itself takes the shape of a quadratic equation (a second-degree polynomial), which has a linearly varying slope.

Note that the quadratic curve can be drawn in two ways. In the diagram below, the curve shown in light blue has a decreasing slope. At the left end of the segment, the curve has a small slope which decreases in value (it becomes more negative) as we approach Point B. The curve shown in dark blue has an increasing slope. Its relatively large negative slope at the left end gets closer to zero as we approach Point B.

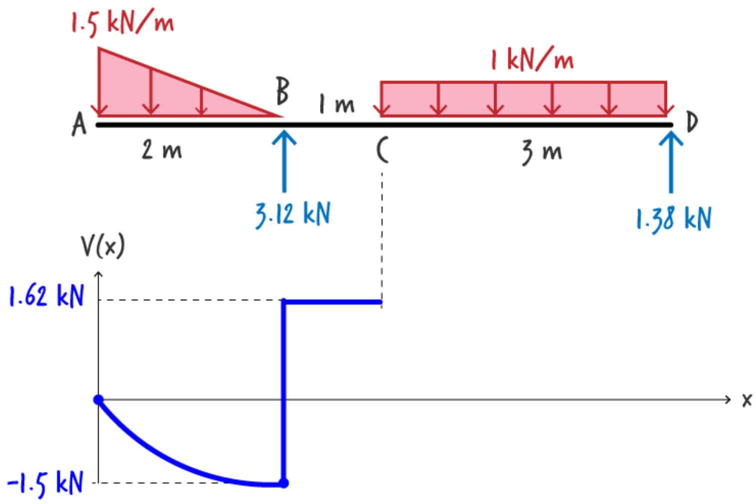


To determine which curve correctly represents the shear diagram, we need to examine the free-body diagram. If the distributed load is increasing in value as we move from A to B, then the curve with an increasing slope is the correct shape. However, if the load is decreasing in value from A to B, then the correct curve has to have a decreasing slope.

The triangular load, since it is downward (negative), has an increasing value from A to B. The load intensity increases from negative 1.5 kN/m to zero as we approach Point B. Therefore, the shear diagram takes the shape of the dark blue curve, as shown below.

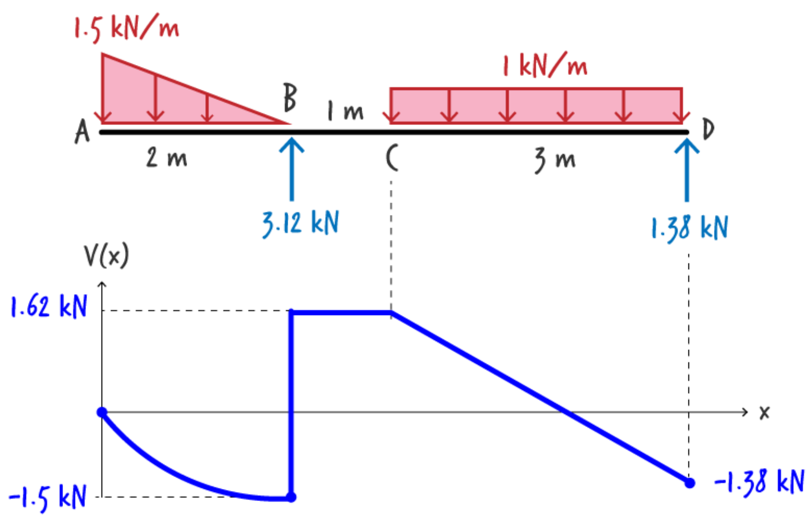


At B, there is going to be a jump of 3.12 kN in shear value due to the upward force present at that point. This means shear at the right side of B is 1.62 kN. Shear remains constant throughout BC since the segment is not subjected to any loads.



As the above diagram indicates, shear at the left side of Point C equals 1.62 kN. And since there is no concentrated force present at C, shear to the right of the point is also 1.62 kN.

Since CD is subjected to a uniformly distributed load, then the area under the load diagram represents the difference in shear values between the two ends of the segment. The area of the rectangle representing the load is (1 kN/m)(3 m) = 3 kN. Therefore, shear at the right end of the segment can be written as: 1.62 – 3 = -1.38 kN. And since the load is uniformly distributed, we simply connect the two end points using a straight line, as shown below.



Finally, since there is an upward concentrated force of 1.38 kN at D, shear jumps from negative 1.38 kN to zero at that point.

The complete shear diagram for the beam is shown below.