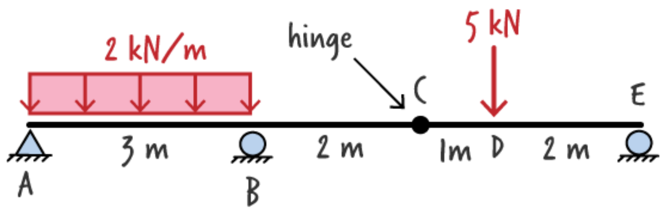
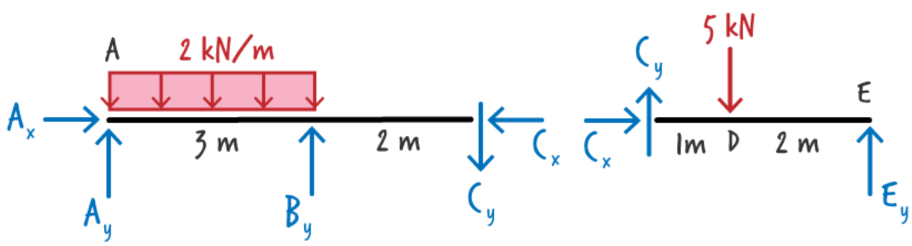
Statics– ST11 (Solution for Exercise Problem 3)
Shear Diagram for Statically Determinate Beams

Draw the shear diagram for the statically determinate beam shown below.

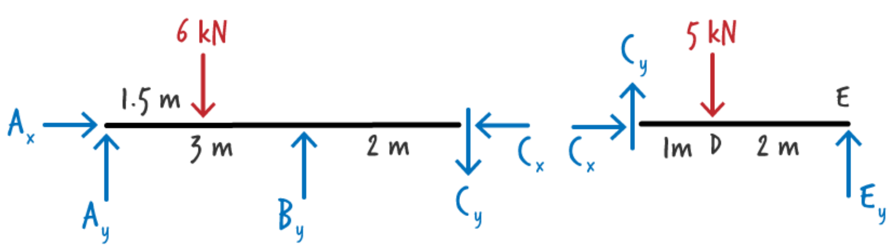


Solution

Draw the beam's free-body diagram. Since there is an internal hinge in the beam, the diagram consists of two parts: segments AC and CE.



Replace the distributed load with its equivalent concentrated load. The magnitude of the concentrated load equals the area of the rectangle representing the load, and the location of the load is the geometric center of the load area.



Now, write and solve the static equilibrium equations for the unknown support reactions. Here, we need to write two sets of equations, one set for each segment.

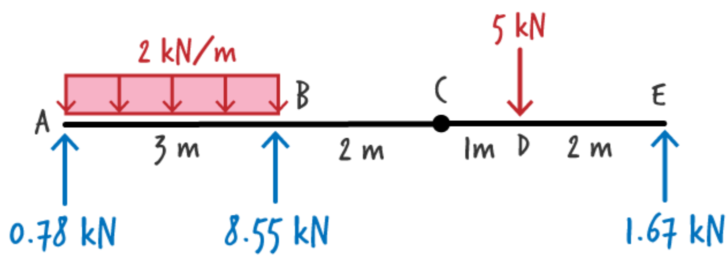
$$\sum F_x = A_x - C_x = 0$$
$$\sum F_y = A_y + B_y - C_y - 6 = 0$$
$$\sum M_{@A} = (1.5)(6) - 3B_y + 5C_y = 0$$

$$\sum F_x = C_x = 0$$
$$\sum F_y = C_y + E_y - 5 = 0$$
$$\sum M_{@C} = (1)(5) - 3E_y = 0$$

Solving the above equations for the unknown forces, we get:

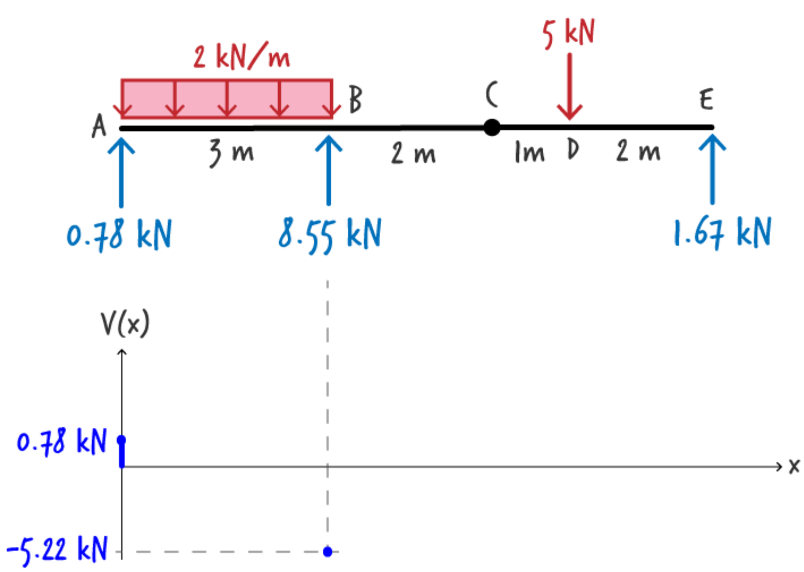
$$A_x = 0$$
$$A_y = 0.78 \text{ kN}$$
$$B_y = 8.55 \text{ kN}$$
$$C_x = 0$$
$$C_y = 3.33 \text{ kN}$$
$$E_y = 1.67 \text{ kN}$$

Knowing the support reactions, now draw the complete free-body diagram for the beam using the distributed load.

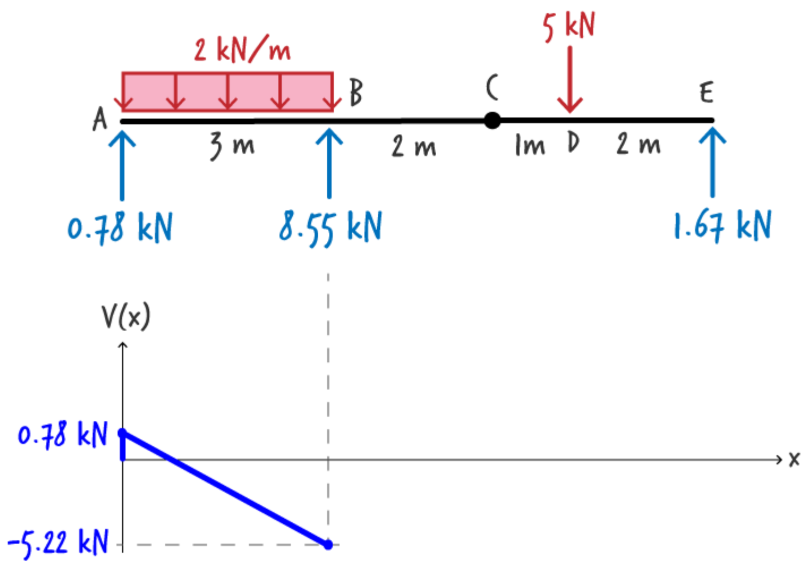


To draw the shear diagram, we start from the left end of the beam. Shear at the left end of AB goes from zero to 0.78 kN, since there is an upward concentrated force with that magnitude at A.

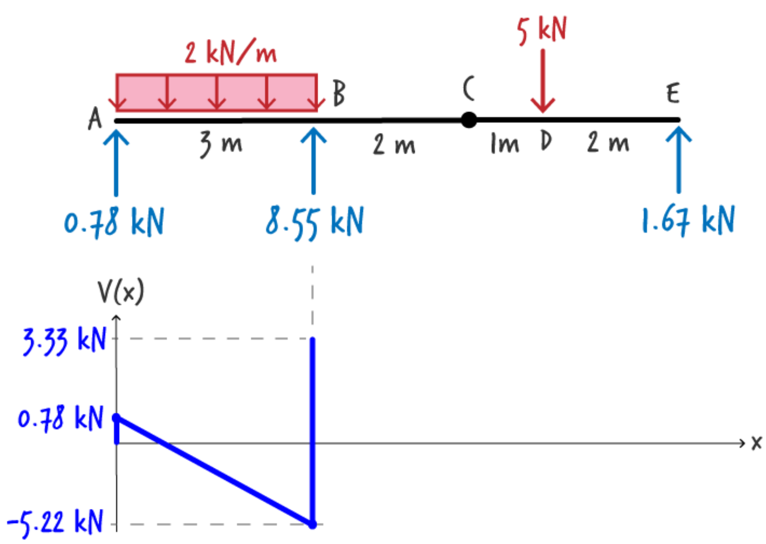
Shear at the right end of AB is equal to the shear at the left end minus the area of the rectangle representing the distributed load. The area is (2 kN/m)(3 m) = 6 kN. So, shear at the right end of AB equals 0.78 – 6 = -5.22 kN. These end shear values are shown in figure below.



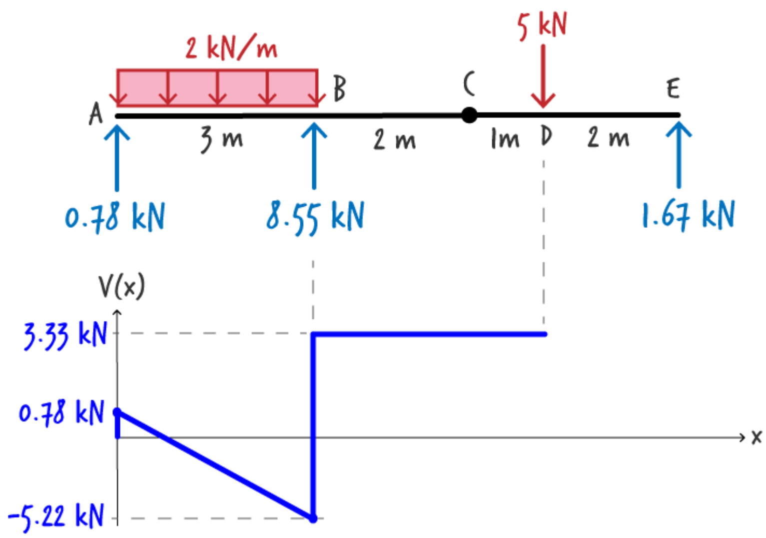
Since the distributed load acting on AB is constant, the shear diagram is going to be a straight line in the segment, as shown below.



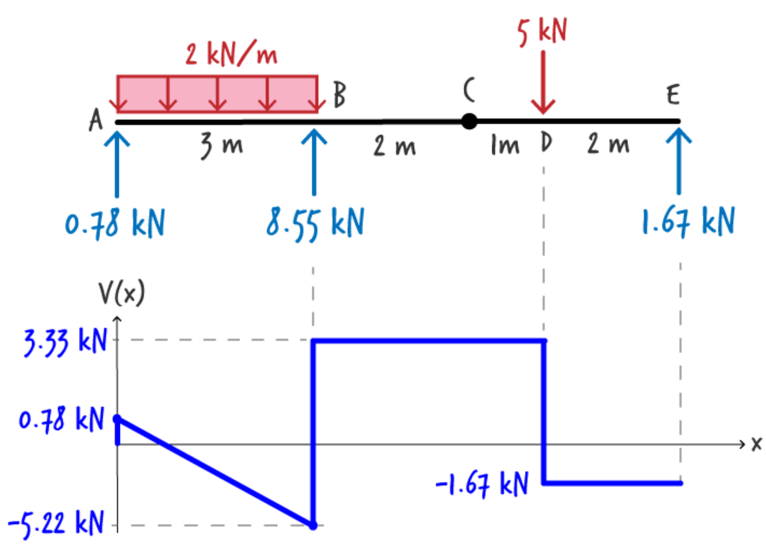
The upward concentrated force at B causes a jump in the shear value. The amount of increase is equal to the magnitude of the upward force; it is 8.55 kN. So, shear jumps from negative 5.22 kN to positive 3.33 kN.



Then, since no load is acting on segments BC and CD, shear remains constant throughout that length of the beam.

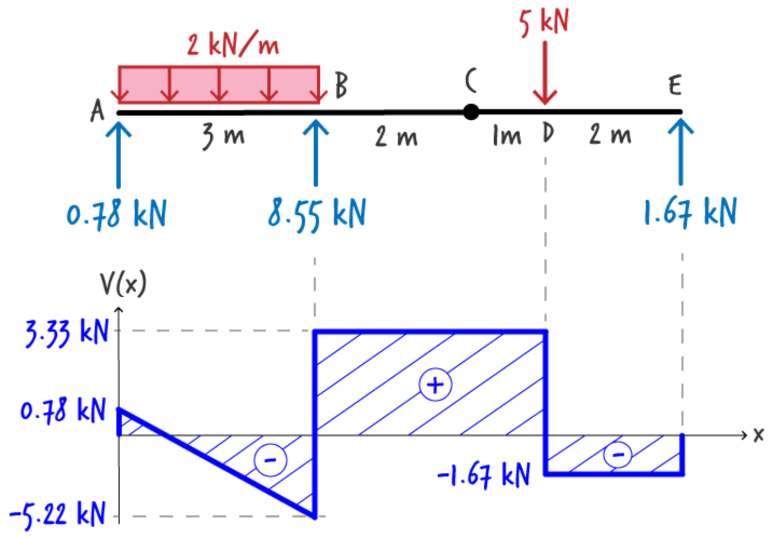


The downward force of 5 kN at D causes a drop in shear value by that amount. So, shear drops from positive 3.33 kN to negative 1.67 kN at D. Then, since DE is not subjected to any external loads, shear remains constant throughout that segment.



Finally, shear returns to zero at E, since there is an upward reaction force of 1.67 kN at the right end of the beam.

The complete shear diagram for the beam is shown below.


2 kN/m
5 kN
A
B
C
E
D
3 m
2 m
1m
2 m
0.78 kN
8.55 kN
1.67 kN
V(x)
3.33 kN
0.78 kN
-5.22 kN
-1.67 kN
x