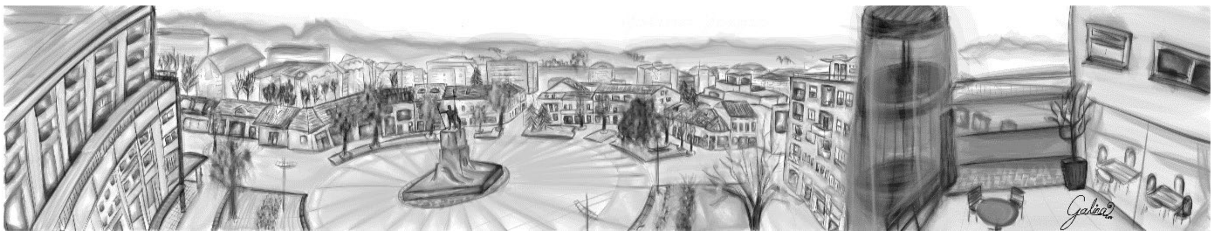
Structural Analysis

Lecture Series



SA01U: Statically Determinate Beams

This document is a written version of video lecture SA01U, which can be found online at the web addresses listed below.

Educative Technologies, LLC

Lab101.Space
https://www.youtube.com/c/drstructure

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2019

Structural Analysis – SA01U

Statically Determinate Beams: An Introduction

This is an introductory lecture on the analysis of statically determinate structures.

A beam is a properly supported structural member capable of carrying loads that are perpendicular to its longitudinal axis, as shown in Figure 1. An applied load could be concentrated at a specific point (also called a point load), or it could be distributed over a portion of the beam.

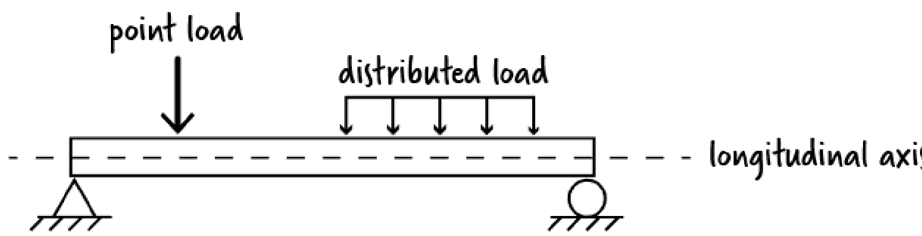


Figure 1: A beam subjected to a point load and a uniformly distributed load

Generally, beams are assumed to bend when they are loaded. Although the amount of bending is often very small and may not be visible to the naked eye, for illustration purposes we draw them in an exaggerated manner, as shown in Figure 2.

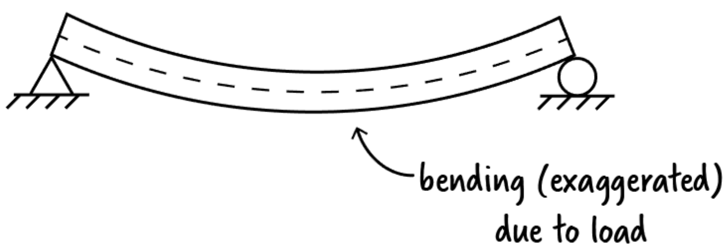


Figure 2: Bending of a beam due to applied load(s)

Beams need to be properly supported in order to be able to carry their loads. The most common types of supports are the pin, roller, and fixed supports (see Figure 3).

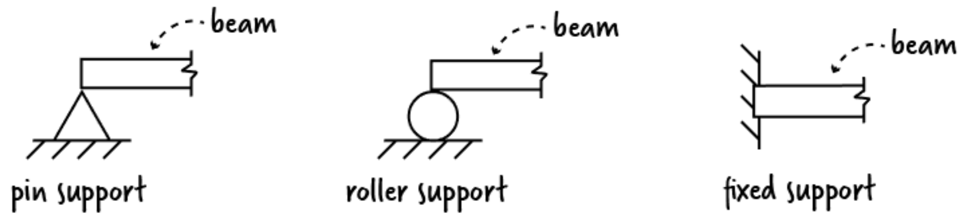


Figure 3. Common support types in beams

How do each of the different types of supports interact with the beam? Each type of support exerts one or more forces on the beam which are called support reactions.

A pin support provides two reaction forces: one in the x-direction and one in the y-direction (see Figure 4a). A roller support has only one reaction force, which is always perpendicular to its surface of contact with the ground, as shown in Figure 4b. Finally, a fixed support provides three reaction forces: one in the x-direction, one in the y-direction, and a bending moment about the z-axis, which is perpendicular to the two-dimensional surface represented by the x-y plane (see Figure 4c).

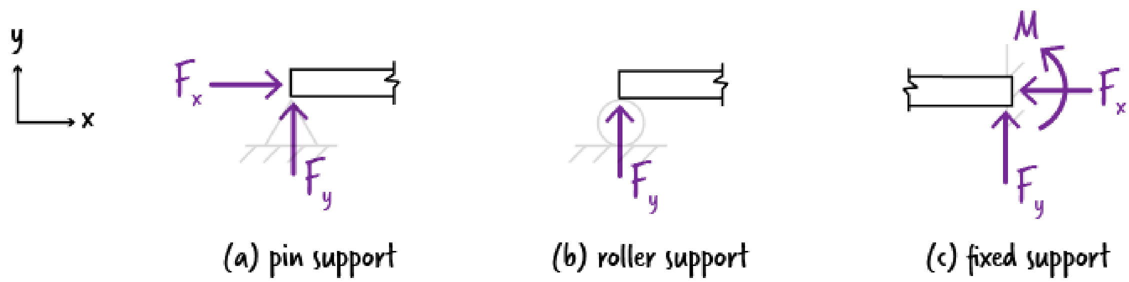


Figure 4: Reaction forces associated with each support type

Consider the simply supported beam shown in Figure 5. It rests on a pin and a roller support. Therefore, three support reactions exist. There are two reaction forces at the pin support, one in the x-direction and one in the y-direction, and there is one reaction force at the roller support in the y-direction.

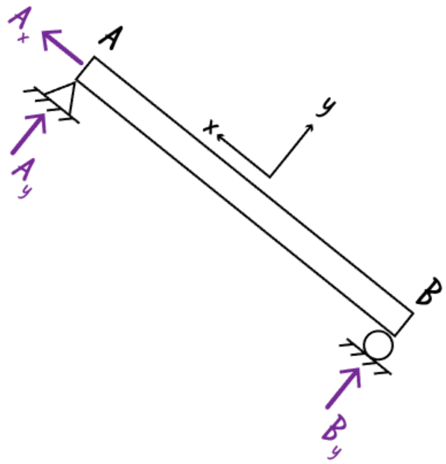


Figure 5: Reaction forces associated with a simply supported beam

Now, consider the cantilever frame shown in Figure 6. There is a fixed support at the lower end of the structure, providing three support reactions: one force in the x-direction, one force in the y-direction, and a bending moment about the z-axis.

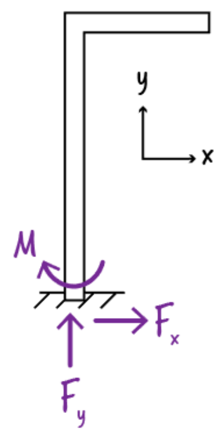


Figure 6: Reaction forces associated with a cantilever frame

The purpose of basic structural analysis is to determine unknown forces in structures. To perform such an analysis, it is customary to draw what is known as a free-body diagram. This diagram shows all the relevant forces acting on the structure and all the necessary distances specified in the Cartesian coordinate system.

Consider the simply supported beam shown in Figure 7a. Its free-body diagram is shown in Figure 7b and shows the applied load and the support reactions, which are all uniquely labeled. In addition, note that the free-body diagram is situated in a coordinate system in which all of the relevant distances have been labeled.

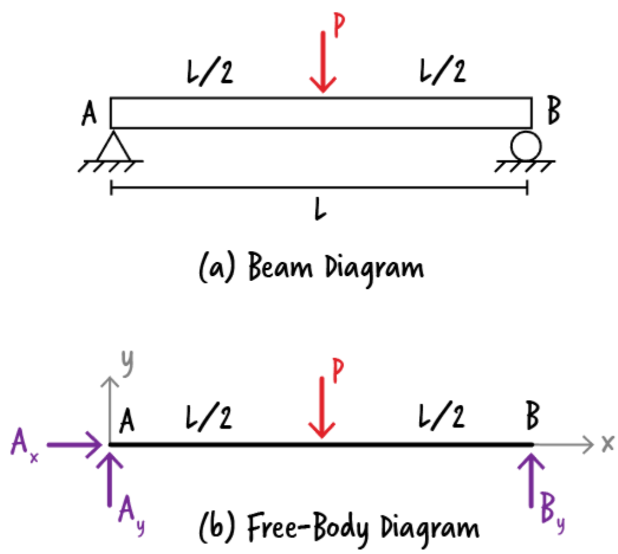


Figure 7: A simply supported beam and its free-body diagram

In this example, for the sake of convenience, we have placed the origin of the coordinate system at the left end of the beam (at point A). However, keep in mind that the origin can be placed anywhere on the x-y plane. The orientation and position of the coordinate system is often chosen in a manner that facilitates the formulation and solution of the equilibrium equations.

The free-body diagram helps us correctly formulate the necessary equilibrium equations, which can then be solved for the unknown support reactions. For structures defined in a two-dimensional space to remain in the state of static equilibrium, three conditions must be satisfied. We refer to these three conditions as the static equilibrium conditions, and they're often written in the form of equations. The static equilibrium conditions are:

1. The sum of the forces in the x-direction must be zero: $\sum F_x = 0$.
2. The sum of the forces in the y-direction must be zero: $\sum F_y = 0$.

3. The sum of the moments about the z-axis taken at any point, such as point A, must be zero: $\sum M_{@A} = 0$.

By applying these equations to the free-body diagram shown in Figure 7b, we get:

$$\xrightarrow{+} \sum M_{@A} = 0 \quad [1]$$

$$+\uparrow \sum F_y = A_y + B_y - P = 0 \quad [2]$$

$$\circlearrowright + \sum M_A = P(L/2) - B_y L = 0 \quad [3]$$

These three equations can be solved for the unknown reaction forces. Solving Equation [1], we get: $A_x = 0$. Equation [3] can then be solved to give: $B_y = P/2$. Finally, Equation [2] can be solved and you find that $A_y = P/2$.

Now, let us consider a cantilever beam subjected to a uniformly distributed load, as shown below.

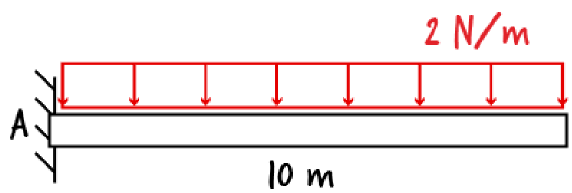


Figure 8: A cantilever beam subjected to a uniformly distributed load

To analyze the beam, we start by drawing its free-body diagram, which shows the uniformly distributed load as well as the support reactions (see Figure 9a). We then replace the uniformly distributed load with its equivalent concentrated load, as depicted in Figure 9b. This transformation is essential for correctly formulating the equilibrium equations since the equilibrium equations refer to point loads and moments.

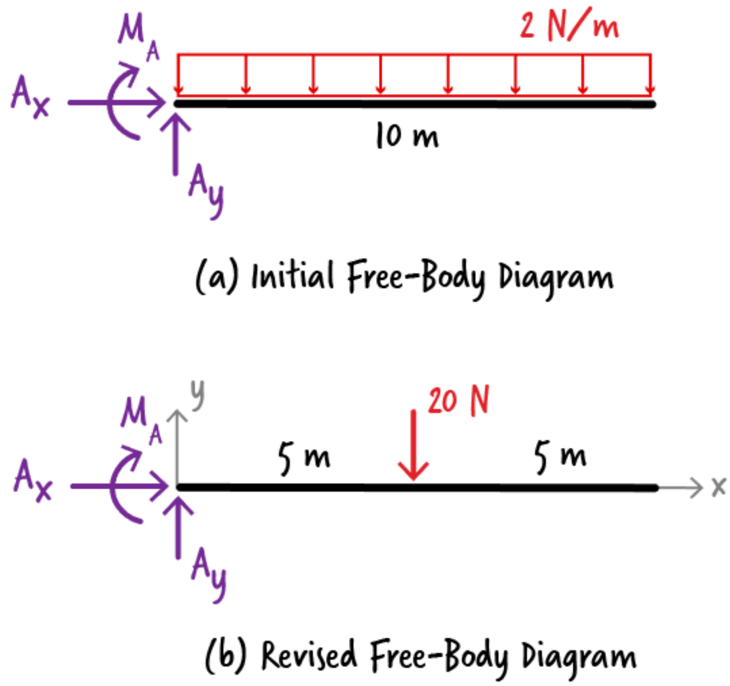


Figure 9: The free-body diagram of a cantilever beam subjected to a uniformly distributed load

Note that the three support reactions at A correspond to those of a fixed support. Analyzing this beam involves calculating the three reaction forces. The equilibrium equations are provided below.

$$\xrightarrow{+} \sum F_x = A_x = 0 \quad [4]$$

$$+\uparrow \sum F_y = A_y - 20 = 0 \quad [5]$$

$$\circlearrowleft + \sum M = M_A + 20(5) = 0 \quad [6]$$

When solved, Equation [4] gives us: $A_x = 0$. The second equilibrium equation, Equation [5] yields: $A_y = 20\ N$. And after solving Equation [6] we get: $M_A = -100\ N\text{-}m$. Notice that the three equilibrium equations allowed us to solve for the three unknowns on this cantilever beam.

Finally, let's consider the simply supported beam shown in Figure 10. It rests on a pin and two roller supports.

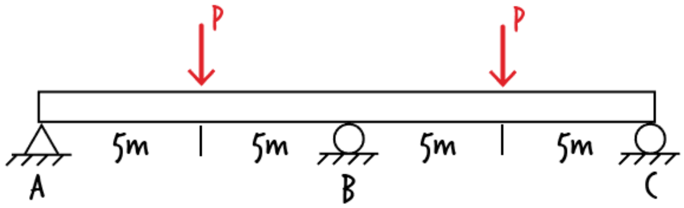


Figure 10: A beam with three supports

The free-body diagram for the beam is drawn in Figure 11.

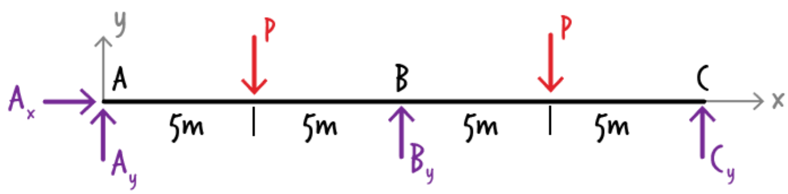


Figure 11: The free-body diagram for a beam with three supports

The free-body diagram shows four unknown reaction forces: two forces at the pin support (A_x, A_y), one force at the middle roller support (B_y), and another force at the right roller support (C_y). The three static equilibrium equations for the beam are as follows:

$$\overset{+}{\rightarrow} \sum F_x = A_x = 0 \tag{7}$$

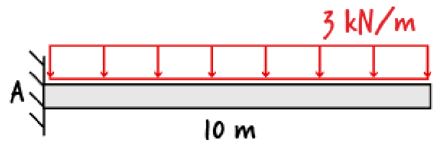
$$+\uparrow \sum F_y = A_y + B_y + C_y - 20 = 0 \tag{8}$$

$$\circlearrowleft + \sum M = 5P - 10B_y + 15P - 20C_y = 0 \tag{9}$$

Here, we have three equations, but four unknowns. Since the number of equations is less than the number of unknowns, we cannot determine all of the support reactions using the static equilibrium conditions. In other words, the beam is not statically determinate; it is an indeterminate beam. We will study the analysis of indeterminate beams in future lectures. For now, see if you can analyze the following statically determinate beams.

Exercise Problems: Analyze each of the beams shown below.

1



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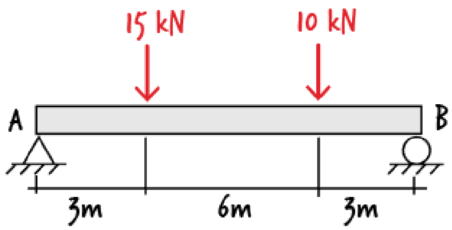

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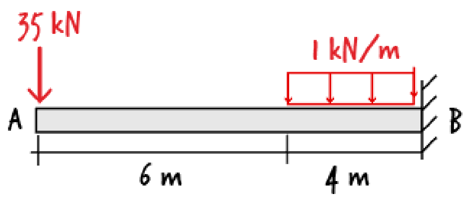

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