# Structural Analysis

## Lecture Series

## SA17: Shear Influence Line for Statically Determinate Beams

This document is a written version of video lecture SA17, which can be found online at the web addresses listed below

### Educative Technologies, LLC

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# Structural Analysis – SA17
# Shear Influence Line for Statically Determinate Beams

This lecture presents a qualitative approach to constructing shear influence lines for statically determinate beams.
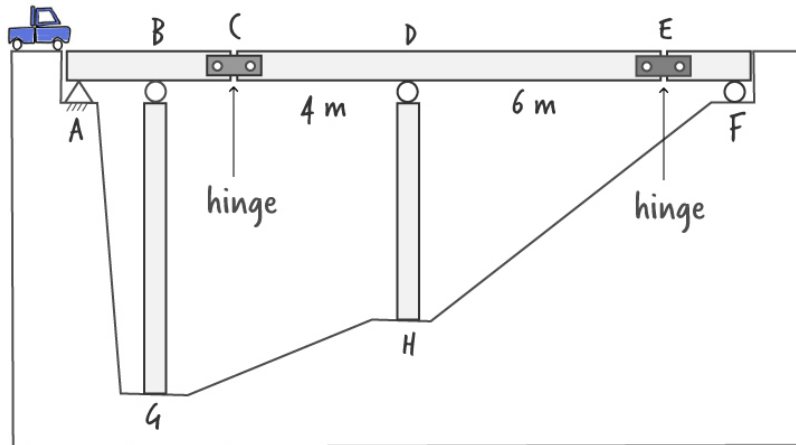


*Figure 1: A statically determinate beam with internal hinges*

Consider the bridge shown in Figure 1. It is subjected to a vehicular (moving) load. Suppose we were to design the hinge at point C. This would require a determination of the maximum shear force produced by the moving load at that particular point.

The maximum positive and negative shear force values at point C can be determined with the aid of a shear influence line. This is a diagram that shows the value of the shear force at a specific point as a function of the position of a moving unit load.

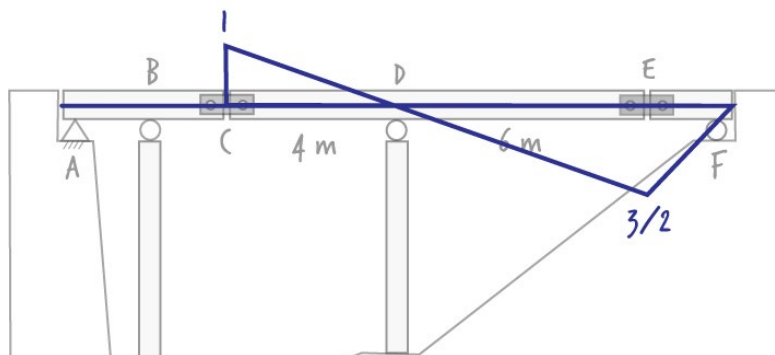The influence line for the shear at point C is shown in Figure 2.



*Figure 2: Shear influence line for point C*

Before we learn how to draw this diagram, let us first understand how to interpret it. As mentioned above, the influence line shows the shear values at point C as a unit load moves across the bridge. For example, when the unit load is to the left of point C, the shear at point C is zero (see Figure 3).
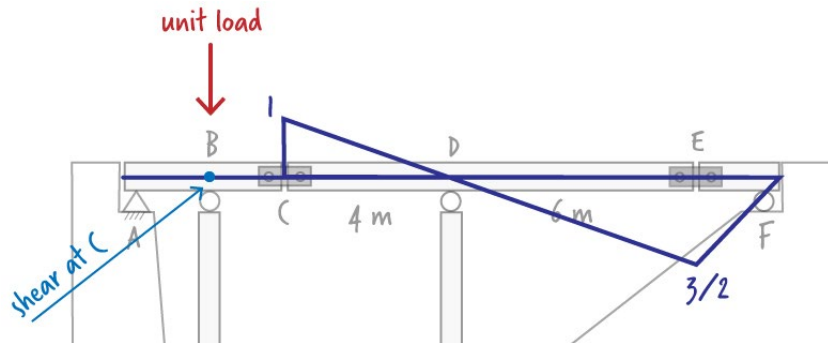


*Figure 3: Reading the shear influence line when the load is just to the left of point C*

When the unit load crosses over point C, the shear at point C jumps from zero to one (see Figure 4).
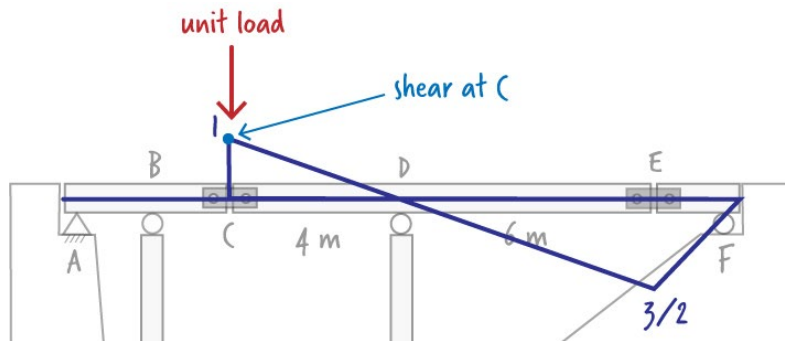


*Figure 4: Reading the shear influence line when the load is at and just to the right of point C*

The shear at point C then continues to decrease linearly as the load continues to move to the right toward point D. When the load reaches point D, the shear at point C becomes zero once again (see Figure 4).

The shear continues to decrease linearly as the load continues to move toward point E. As shown in Figure 5, when the load reaches point E, the shear at point C reaches its maximum negative value of 3/2.
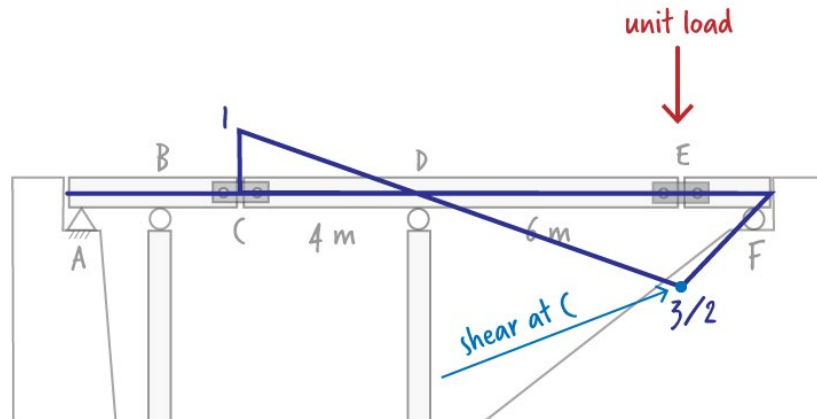
*Figure 5: Reading the shear influence line when the load is at point E*

Once the moving load crosses point E, the shear force begins to increase from negative 3/2 toward zero; it reaches zero when the unit load reaches point F.

Please keep in mind that since the influence line is drawn for a unit load, we must multiply the values given by the diagram by the actual magnitude of the load in order to obtain the correct shear value. For example, if the moving vehicle exerts a force of 6 kN on the bridge, then the maximum negative shear force at point C (when the unit load is at point E) equals: (6 kN)(3/2), or 9 kN (see Figure 6). Therefore, the hinge at C will need to be designed for a force of 9 kN.

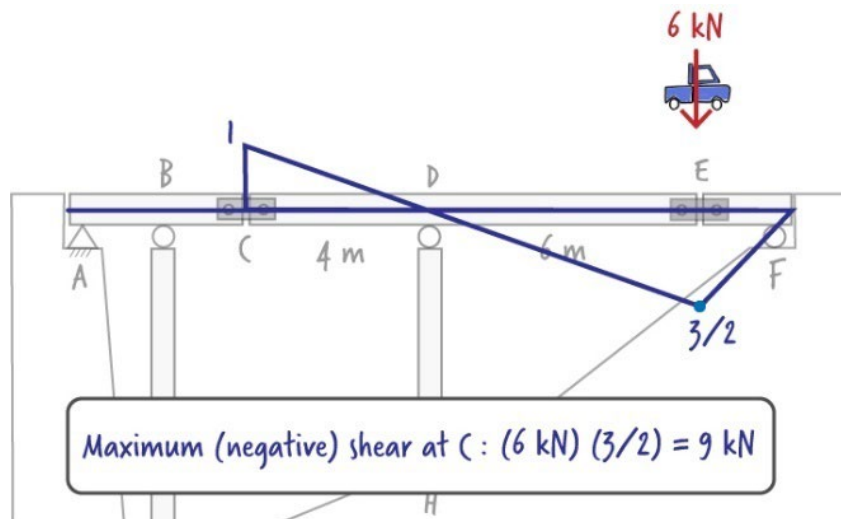

Maximum (negative) shear at C : (6 kN) (3/2) = 9 kN

*Figure 6: Maximum negative shear force at hinge C due to a vehicular load*

Consider the simply supported beam shown in Figure 7. Suppose we wish to draw the influence line for shear at point C. That is, we want to show the shear values at point C as a unit load moves across the beam.
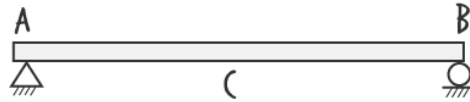
*Figure 7: A simply supported beam*

If we place the unit load at various locations on the beam and calculate the shear force at point C for each load position, we can then plot the shear values as a function of the load position to create a graph similar to that shown in Figure 8.
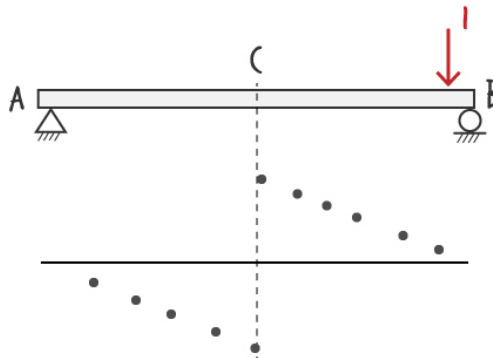


*Figure 8: The plot of shear values for point C*

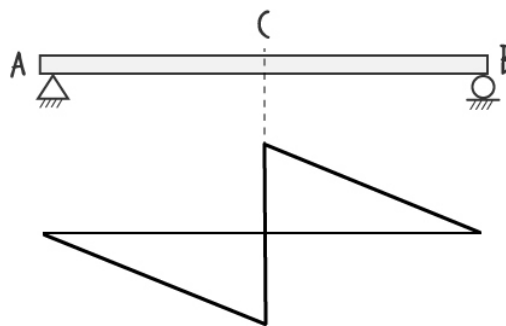If we connect these points with straight lines, we get the shear influence line (see Figure 9).



*Figure 9: The shear influence line for point C*

Figure 9 tells us that the shear at point C decreases linearly from zero to a negative value as the unit load moves from point A to point C. At point C, there is a jump in the shear value from negative to positive. This means that when the unit load is just to the left of point C, the shear at point C is negative. When the unit load moves just to the right of point C, the shear at point C becomes positive. The shear at point C then decreases linearly again toward zero as the load moves from point C to point B.

The shear diagram consists of two line segments separated at point C. It is as if the beam were cut at point C with the left side pushed down and the right side pushed up, as points A and B rotate to accommodate the movement. The idea of pushing the beam segments up or down and drawing the resulting shape is a convenient way to draw shear influence lines qualitatively.

To better apply this qualitative approach for drawing shear influence lines, we can imagine a pair of vertical rollers being placed at point C, permitting vertical movement at that point (see Figure 10).
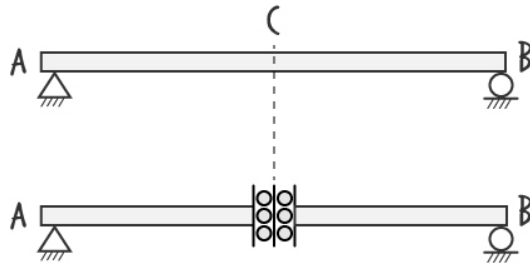


*Figure 10: Imaginary vertical rollers at point C*

We then apply a positive shear force to the rollers. A positive shear in beams means having a downward force applied to the left roller and an upward force applied to the right roller, as shown in Figure 11.
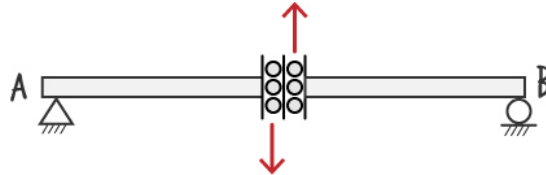


*Figure 11: A positive shear force placed at point C*

These forces are imagined to have the ability to move the two rollers in opposite directions. The downward shear force causes a downward movement of the left roller, and the upward shear force enables the right roller to move up, as depicted in Figure 12.
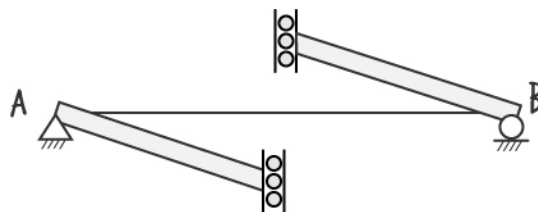


*Figure 12: Beam segments' rotation due to an imaginary pair of shear forces*

Next, we draw the centerlines of the two beam segments and connect them at point C, thereby rendering a line which we refer to as the influence line for shear at point C (see Figure 13).
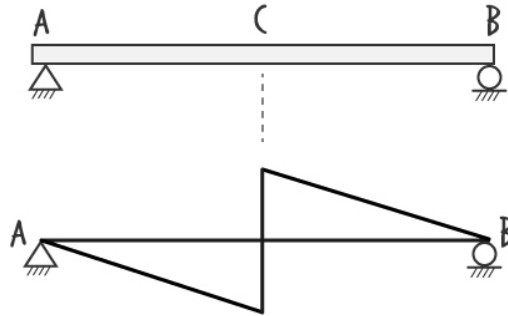


*Figure 13: Shear influence line for point C*

Now let's draw the influence line for shear at point C for the beam shown in Figure 14, which has an overhang.



*Figure 14: A statically determinate beam with an overhang*

First, we place a pair of imaginary vertical rollers at point C, as shown in Figure 15. Then we apply a positive shear force to the rollers.
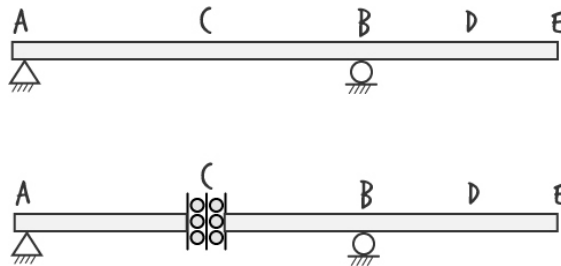


*Figure 15: A pair of imaginary vertical rollers placed at point C*

The rollers divide the beam into two rigid segments, which we will call bars. We have bar AC and bar CE. How will these bars deform?

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To determine the displaced shape of the bars, it is very important to keep in mind that they can move up and down or rotate if permitted by their boundary conditions, but they cannot bend. As such, any deformation resembling that shown in Figure 16 is NOT permissible.
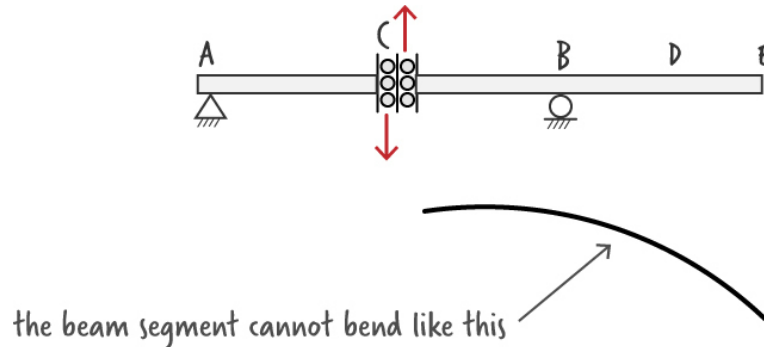


the beam segment cannot bend like this

*Figure 16: An invalid deformation of a beam segment due to a shear force*

To determine the displaced shape of bar AC, we observe that point C is being pushed down by a shear force. Since the pin at A permits the bar to rotate, AC will have a clockwise rotation, as shown in Figure 17.
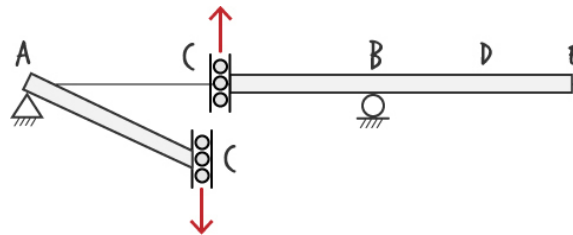


*Figure 17: Displacement of bar AC due to the applied shear force at point C*

Under the same shear force, how will bar CE displace? Here, the left end of the bar is being pushed up by the applied shear force at point C. Given that the roller at point B permits rotation (a pin or a roller support can rotate) and end E is free, it follows that the upward movement of point C will make the entire bar rotate in a clockwise direction around B, as shown in Figure 18.
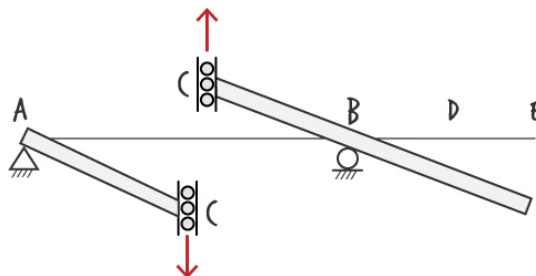


*Figure 18: Displacement of bar CE due to the applied shear force*

By drawing the centerlines of the two beam segments and connecting them together at C, we get the shear influence line shown in Figure 19.
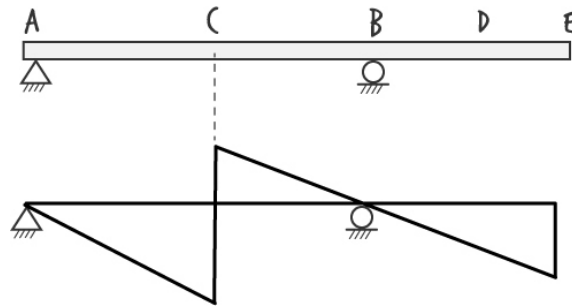


*Figure 19: Influence line for shear at point C*

Now let's draw the influence line for shear at point D for the beam shown in Figure 14. First, we place an imaginary pair of vertical rollers at point D. Then we subject the rollers to a positive shear force, as shown in Figure 20.
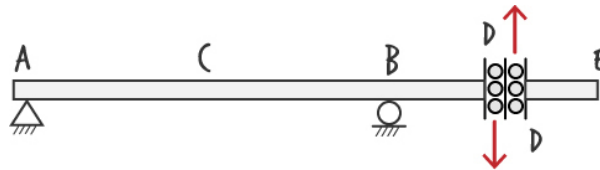


*Figure 20: A pair of imaginary vertical rollers placed at point D*

Now we draw the deformed shape of the beam. The rollers divide the beam into two bars: AD and DE. Under this shear force, bar DE moves up vertically, as shown in Figure 21. But why does the entire beam move up? Since there are no support points in segment DE to enable rotation, both ends of the segment are free to move up.
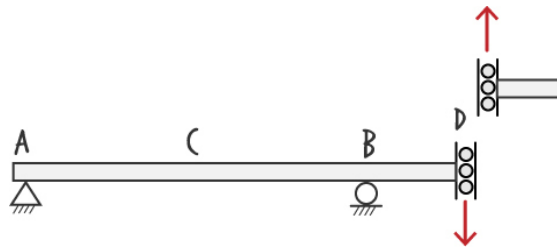


*Figure 21: Displacement of bar DE due to a shear force at point D*

Bar AD, however, is not going to have any deformation at all. While it is true that point D is being pushed down, the point would move if, and only if, the bar was allowed to rotate around point B. For the bar to rotate at point B, point A would have to move up. However, since point A is pinned, it will not be allowed to vertically displace. As such, bar AD cannot move as much as

it wants to do so. Therefore, the entire influence line for shear at point D has a nonzero area only to the right of point D, as shown in Figure 22.
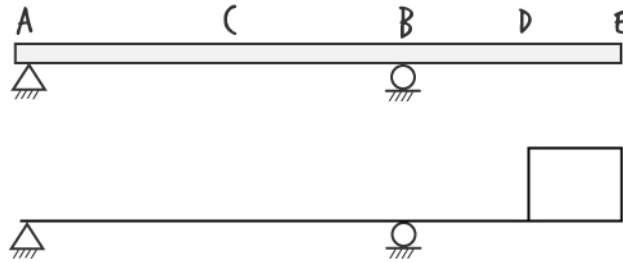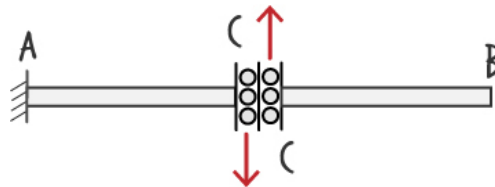


*Figure 22: Influence line for shear at point D*

Now let's consider the cantilever beam shown in Figure 23.



*Figure 23: A cantilever beam*

We wish to draw the influence line for the shear force at point C. First, we place a pair of imaginary vertical rollers at point C and then we subject the rollers to a positive shear force, as shown below.



*Figure 24: A pair of imaginary vertical rollers placed at point C*

Now we draw the displaced shape that results. The beam is divided into two bars: AC and CB. Bar CB moves up since both points C and B are free to move vertically. Bar AC, however, remains undeformed since A is fixed and cannot move or rotate (see Figure 25).
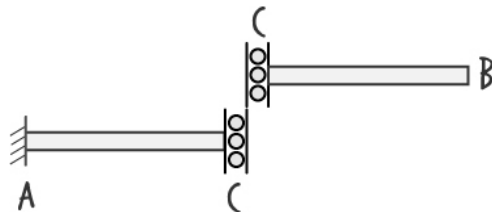


*Figure 25: Displacement of bar CB due to a shear force at point C*

The complete influence line for the shear at point C is shown in Figure 26.
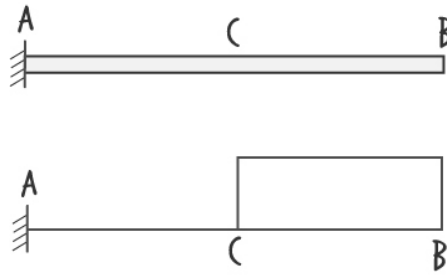


*Figure 26: Influence line for shear at point C*

Now let's consider beams with one or more hinges. Figure 27 depicts a statically determinate beam with an internal hinge at point D.



*Figure 27: A beam with an internal hinge at point D*

Suppose we wish to draw the shear influence line for point E. We start by placing a pair of vertical rollers at point E and then subject them to a positive shear force, as shown in Figure 28.
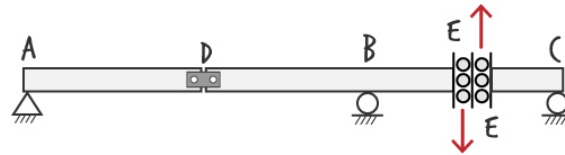


*Figure 28: A beam with an internal hinge subjected to a shear force at point E*

The hinge and the rollers divide the beam into three bars: AD, DE, and EC. For bar EC, the upward force pushes point E up as the entire bar rotates around point C, as shown in Figure 29.
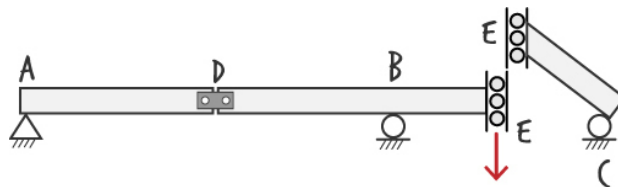


*Figure 29: Displacement of bar EC due to a shear force at point E*

For bar DE, we have a force pushing down at E. This means that the beam has to rotate clockwise at point B, which forces point D to move up. For point D to have an upward displacement, bar AD must then rotate counterclockwise, as shown in Figure 30.
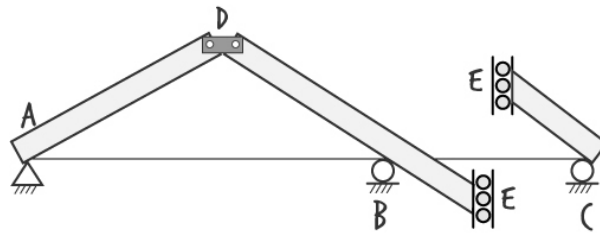


*Figure 30: Displacement of bars AD and DE due to a shear force at point E*

Finally, we draw the centerlines of each bar and connect them at point E, which gives us the influence line for shear at point E (see Figure 31).
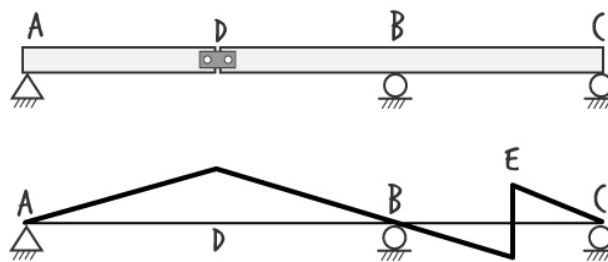


*Figure 31: Influence line for shear at point E*

Now let's draw the influence line for the shear at hinge D for the beam shown in Figure 27. We start by placing an imaginary pair of vertical rollers at point D, and then we apply a unit shear force to the rollers, as shown in Figure 32.
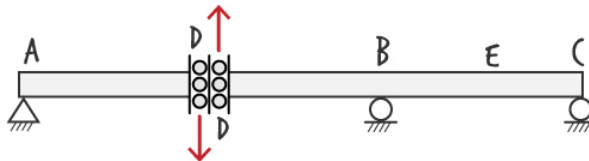


*Figure 32: A pair of imaginary vertical rollers placed at point D*

The downward shear force at point D wants to rotate bar AD in a clockwise direction. Since the pin at A permits rotation, the bar does indeed rotate, accommodating the downward displacement. The upward shear force on the left end of bar DC wants to lift end D, but for point D to move up, the bar would need to be able to rotate around point B. For that to happen, point C would need to move down, but the roller at point C prevents any vertical displacement. Therefore, no rotation at point B nor upward movement at point D can occur (see Figure 33).
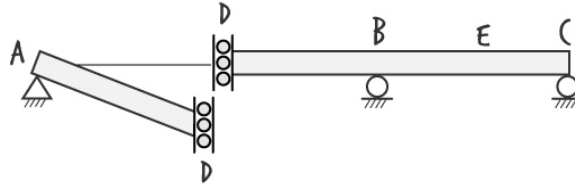


*Figure 33: Displacement of bar AD due to a shear force at point D*

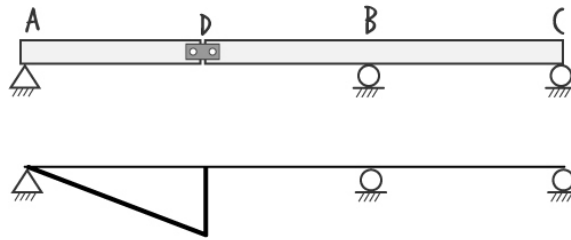Therefore, the shear influence line for point D can be drawn in the following manner:



*Figure 34: Influence line for shear at point D*

Now let's look at an example involving two hinges (see Figure 35).
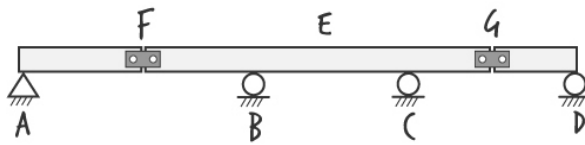


*Figure 35: A beam with two internal hinges*

What does the influence line for shear at point E look like? Let's start by placing a pair of imaginary vertical rollers at point E and subjecting it to a positive shear force, as shown in Figure 36.
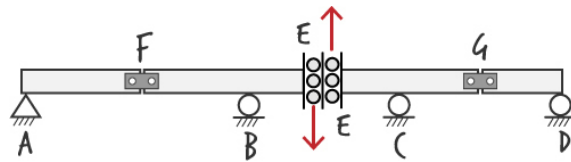
*Figure 36: A pair of imaginary rollers placed at point E*

Now we can draw the resulting deformed shape of the beam. Here the beam is divided into four bars: AF, FE, EG, and GD. Due to the upward shear force at point E, bar EG wants to rotate clockwise. However, it can do so only if point G (one of the internal hinges) is permitted to move down in segment GD. Since there is a roller at point D, bar GD can rotate. The deformed shape of bars EG and GD are as shown in Figure 37.
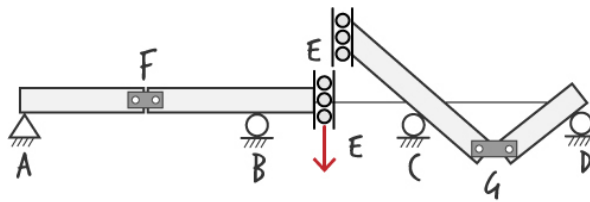


*Figure 37: Displacement of bars EG and GD due to a shear force at point E*

Similarly, bar FE wants to turn clockwise, since end E is being pushed down. However, the bar can rotate only if the hinge at F is permitted to move up in response. Since there is a pin at point A permitting rotation, hinge F can move up, as depicted in Figure 38.
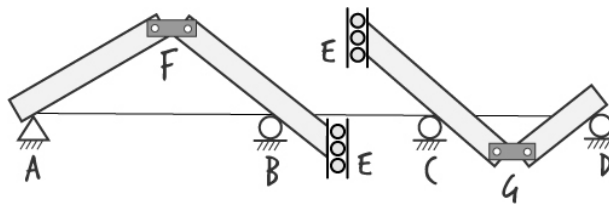


*Figure 38: Displacement of bars AF and FE due to a shear force at point E*

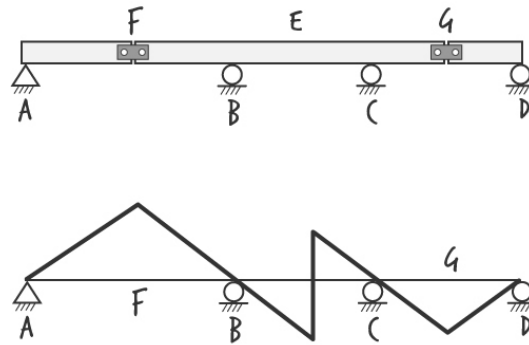The influence line for shear at point E is as shown in Figure 39.



*Figure 39: Influence line for shear at point E*

Now let's talk about determining shear magnitudes in influence lines.

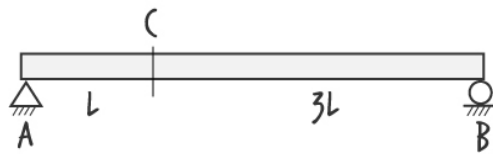Consider the simply supported beam shown in Figure 40.



*Figure 40: A simply supported beam having a length of 4L*

Suppose we wish to draw the influence line for the shear at point C. We start by placing an imaginary pair of vertical rollers at point C and then apply a positive shear force to the rollers, as shown in Figure 41.
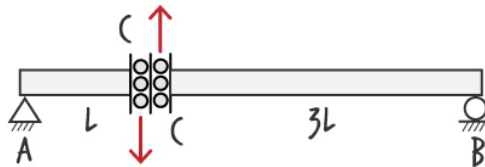


*Figure 41: A pair of imaginary vertical rollers placed at point C*

The rollers divide the beam into two bars (AC and CB), which then displace, as shown in Figure 42.
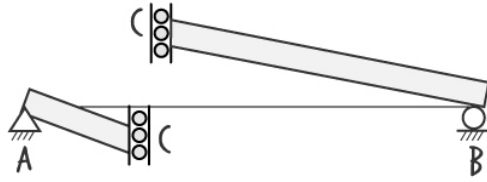
*Figure 42: Displacement of bars AC and CB due to a shear force at point C*
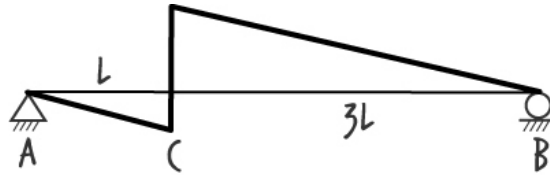
The resulting influence line is as follows:



*Figure 43: Influence line for shear at point C*

Let us refer to the vertical distances from the x-axis to the bottom and top of the influence line at point C as $h_1$ and $h_2$, respectively (see Figure 44).
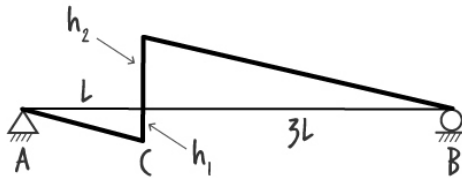


*Figure 44: The heights of influence line at point C*

Since the influence line is drawn for a unit load, the total vertical distance at point C representing the change in shear value is 1, which is expressed as:

$$h_1 + h_2 = 1$$

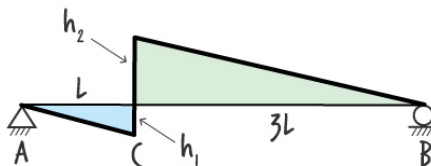Note how the influence line forms two similar triangles (see Figure 45).



*Figure 45: Influence line forming two similar triangles*

We can use similar triangle side ratios to find $h_1$ and $h_2$. That is, $\dfrac{h_1}{L} = \dfrac{h_2}{3L}$, or $h_2 = 3h_1$.

And since $h_1 + h_2 = 1$, we can solve for $h_1$ and $h_2$ as follows: $h_1 = 1/4$ and $h_2 = 3/4$. The precise shape of the influence is as shown in Figure 46.
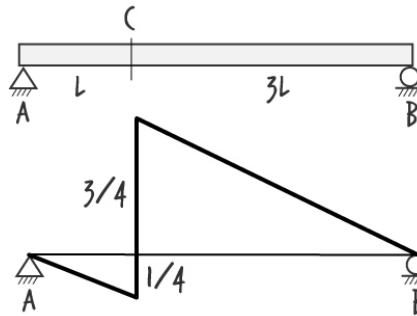


*Figure 46: Influence line for the shear at point C*

As another example, let's consider the beam shown in Figure 47.
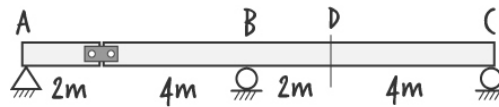


*Figure 47: A beam with an internal hinge*

Suppose we wish to draw the influence line for the shear at point D. We place a pair of vertical rollers at point D, subject the rollers to a positive shear force, and then draw the displaced shape of the beam as shown below in Figure 48.
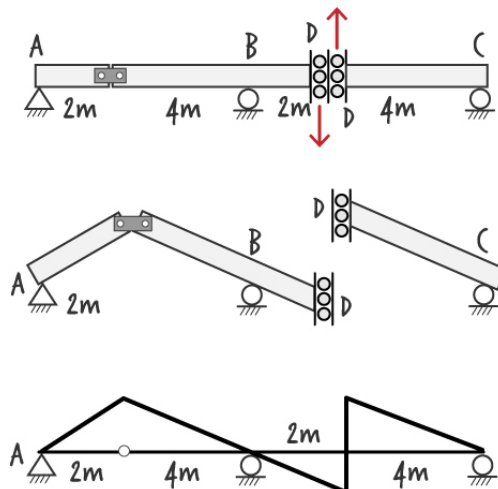


*Figure 48: Influence line for the shear at point D*

The part of the influence line between point B and point C can be viewed as two right triangles. Let's label the two heights at point D as $h_1$ and $h_2$, as shown in Figure 49.
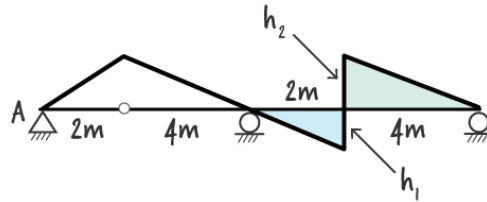


*Figure 49: Influence line forming two similar right triangles*

Using the basic geometric relationships between the two triangles, we can write the following:

$$\frac{h_1}{2} = \frac{h_2}{4}$$

Further, since $h_1 + h_2 = 1$, the two heights can be determined as follows: $h_1 = 1/3$ and $h_2 = 2/3$. The influence line can then be drawn to scale, as shown in Figure 50.
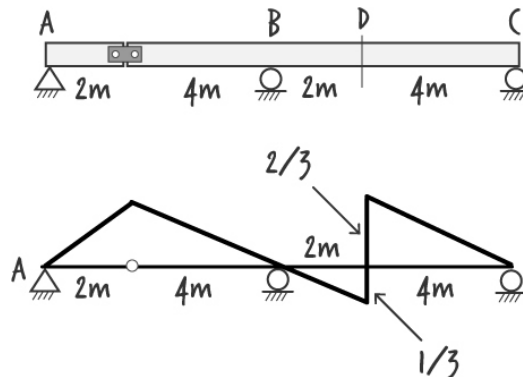


*Figure 50: Influence line for the shear at point D*

We can determine the height of the diagram at the internal hinge using similar triangles as well. Note the colored triangles in Figure 51.
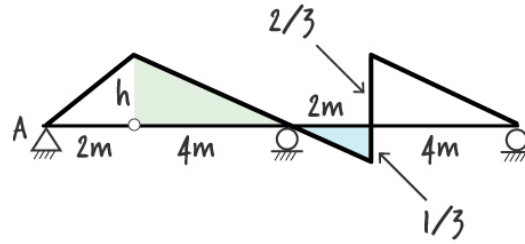


*Figure 51: Similar triangles for the center part of an influence line*

The height-to-base ratio of the two triangles give us: $\dfrac{h}{4}=\dfrac{1/3}{2}$. Therefore, $h=2/3$.

Now let's go back and consider the influence line shown in Figure 26. What is the height of the diagram at point C?

We know that the overall height between the rollers is always one. Since, in this case, there is only one height located on the positive side of the x-axis, that height must be one (see Figure 52).
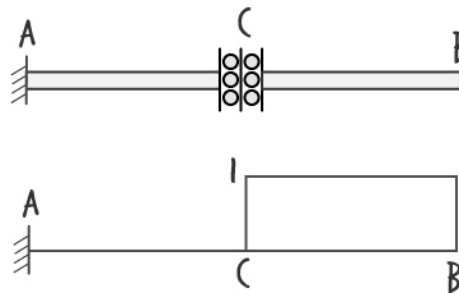


*Figure 52: Influence line for the shear in a cantilever beam*

Let's wrap up this lecture by drawing the influence line for the shear at the hinge in the beam shown in Figure 47. We start by placing a pair of vertical rollers at the hinge (point E), and then apply a positive shear force to the rollers, as shown in Figure 53.
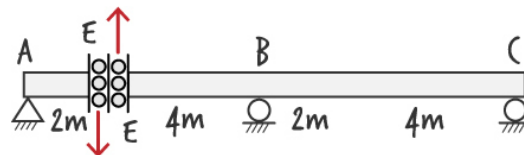


*Figure 53: A pair of vertical rollers and a shear force placed at point E (the hinge)*

Bar AE turns clockwise, since the downward shear force at point E pushes the right end of the beam down, while the left end of the beam rotates around point A. The upward force at point E wants to cause a clockwise rotation of bar EC. For this rotation to happen, the bar would need to rotate around point B. The roller at point C, however, prevents such rotation. Consequently, bar EC remains undeformed, as depicted in Figure 54.
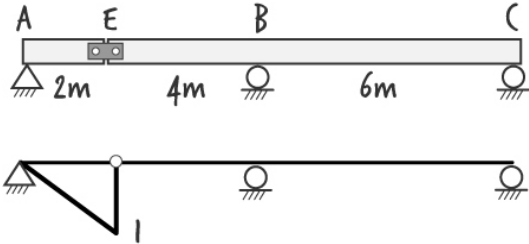


*Figure 54: Influence line for the shear at point E*

As such, the influence line has a nonzero triangular shape only at the left end of the beam (from point A to point E), with a height of one at point E where the hinge is located. The remaining part of the diagram is flat on the x-axis, signifying that the shear at point E remains zero once the unit load moves to the right of the hinge.