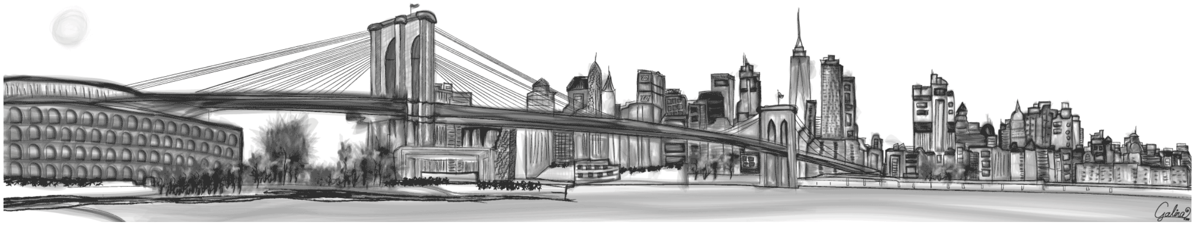
Structural Analysis
Lecture Series



SA63: Cables
Part I: Concentrated Loads

This document is a written version of video lecture SA63, which can be found online at the web addresses listed below.

Educative Technologies, LLC

http://www.Lab101.Space
https://www.youtube.com/c/drstructure

2019

Structural Analysis – SA63

Cables – Concentrated Loads

Consider a cable attached to two poles, one at each end. Hanging from the cable are three concentrated loads. The cable sags under the applied loads, forming a stable configuration in which the conditions of static equilibrium are maintained. Assuming that the horizontal distances between the loads and the support points are known, we can describe the shape of the cable using four heights. These heights are labeled h_1 through h_4 in Figure 1.

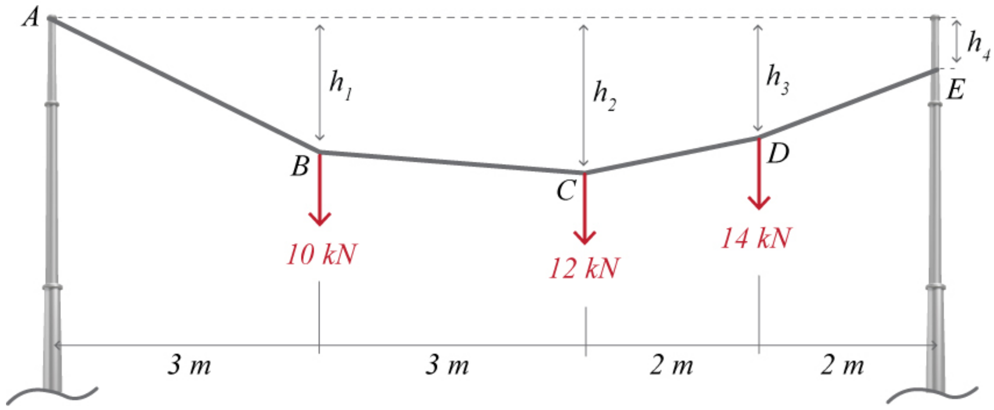


Figure 1: A cable subjected to three concentrated loads

The analysis of such a cable requires calculation of: (1) the reaction forces at *A* and *E*, (2) the tension force in each interior segment of the cable, and (3) the unknown heights, if there are any.

A general approach for analyzing the cable shown in the figure above (or any cable system subjected to a series of concentrated loads) is to determine the reaction forces and unknown heights before determining the tension forces in the cable.

Viewing the two ends of the cable as pin connections, four unknown support reactions are present; there are two unknown forces at each end of the cable (see Figure 2). Additionally, there are up to 4 unknown heights (labeled h_1 through h_4 in the figure). Therefore, in order to completely analyze this cable system, we need to determine up to eight (8) unknowns.

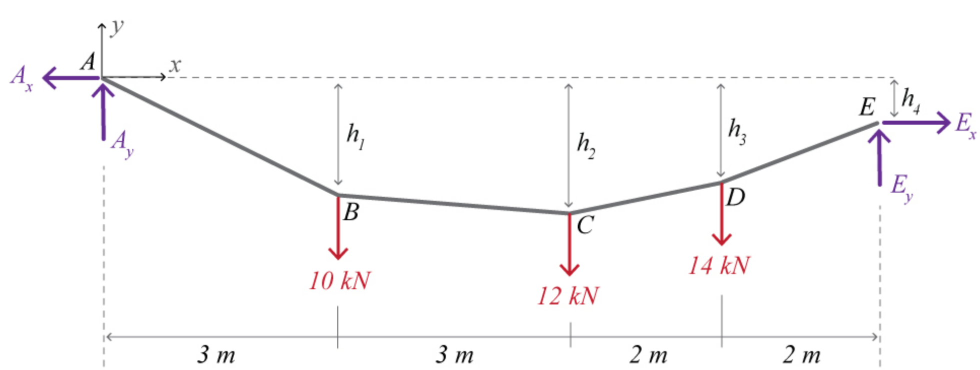


Figure 2: Free-body diagram of a cable subjected to three concentrated loads

Considering the free-body diagram of the entire system, we can write the following three equilibrium equations:

$$\sum F_x = E_x - A_x = 0 \qquad [1]$$
$$\sum F_y = A_y + E_y - 10 - 12 - 14 = 0 \qquad [2]$$
$$\sum M @ A = 10E_y + h_4 E_x - 3(10) - 6(12) - 8(14) = 0 \qquad [3]$$

We can generate three additional equations by setting equal to zero the sum of the moments about each interior joint of the system. For joint B, first cut the cable in segment BC (see Figure 3). Then the equilibrium equation can be written as shown in Equation [4].

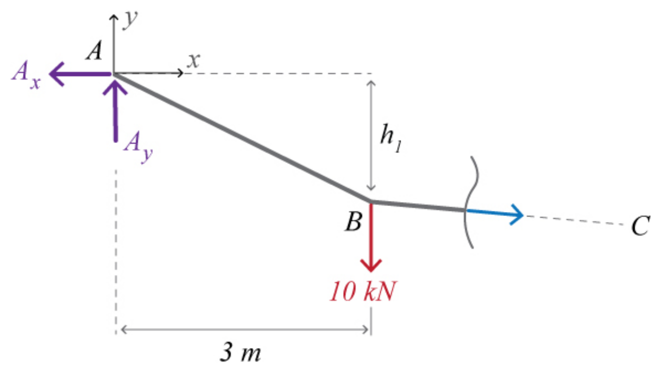


Figure 3: Free-body diagram of a single cable segment

$$\sum M @ B = 3A_y - h_1 A_x = 0 \qquad [4]$$

If we cut the cable in segment CD, the free-body diagram of the system to the left of joint D can be drawn as shown in Figure 4.

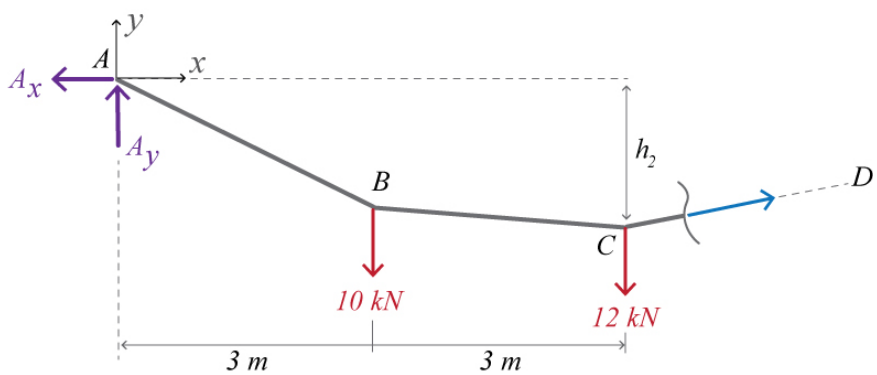


Figure 4: A free-body diagram consisting of two cable segments

The moment equilibrium equation for joint C, then, can be written as follows.

$$\sum M @ C = 6A_y - h_2 A_x - 3(10) = 0 \qquad [5]$$

Cutting the cable in segment DE results in the free-body diagram shown in Figure [5], which yields Equation [6].

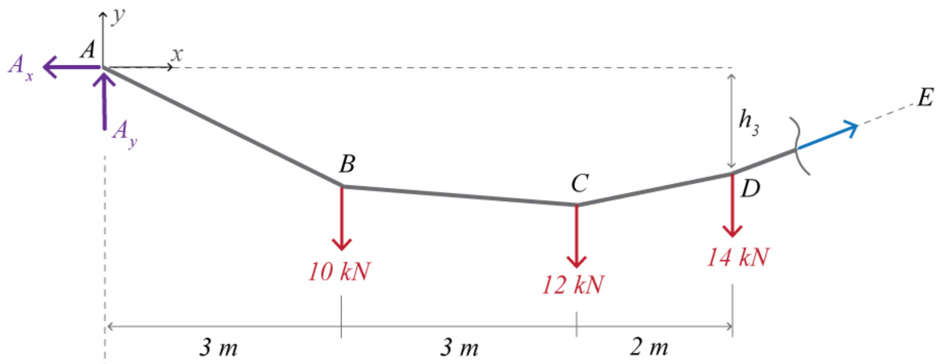


Figure 5: A free-body diagram consisting of three cable segments

$$\sum M @ D = 8A_y - h_3 A_x - 5(10) - 2(12) = 0 \qquad [6]$$

Equations [1] through [6] can be solved simultaneously for six of the eight unknowns. Therefore, we need to know two of the heights in order to be able to solve the equations. Let's assume the maximum amount of sag in the cable and the position of the right support are known. That is, we know h_2 and h_4. Say, $h_2 = 0.5m$ and $h_4 = 0$.

Substituting 0.5 for h_2 and 0 for h_4 in Equations [1] through [6], the following set of equations results:

$$\begin{aligned}
&E_x - A_x = 0 \\
&A_y + E_y - 10 - 12 - 14 = 0 \\
&10E_y - 3(10) - 6(12) - 8(14) = 0 \\
&3A_y - h_1 A_x = 0 \\
&6A_y - 0.5A_x - 3(10) = 0 \\
&8A_y - h_3 A_x - 5(10) - 2(12) = 0
\end{aligned} \qquad [7]$$

The solution to the above system of equations is: $A_x = 115.2\ kN$, $A_y = 14.6\ kN$, $E_x = 115.2\ kN$, $E_y = 21.4\ kN$, $h_1 = 0.38\ m$, $h_3 = 0.37\ m$. The calculated support reactions and heights are depicted in Figure 6.

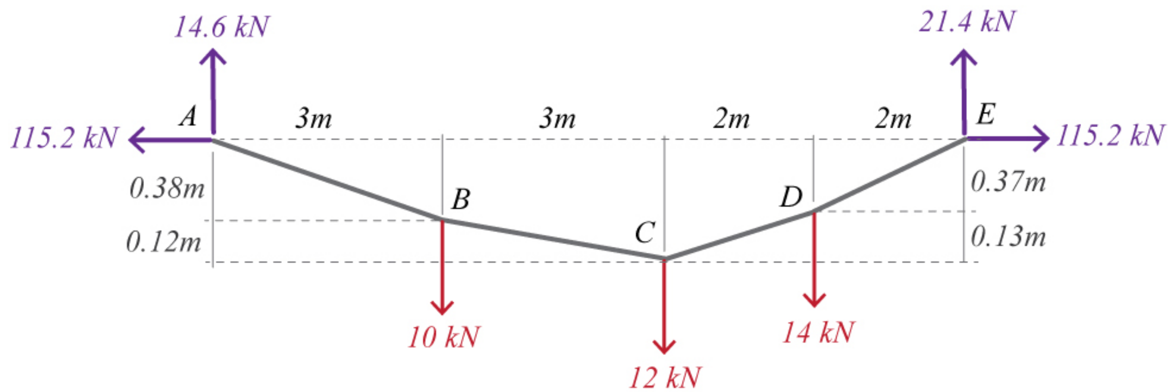


Figure 6: Support reactions and heights in a cable system subjected to three concentrated loads

Knowing the support reactions and all the heights, we are now ready to calculate the tension force in each segment of the cable. For segment AB, we can determine the tension force by finding the vector sum of reaction forces at point A (see Figure 7).

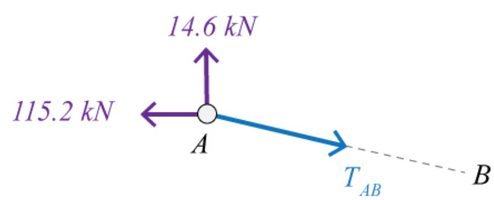


Figure 7: Free-body diagram of a cable joint

For joint A to be in equilibrium, the sum of the forces at the joint must be zero. That is, since the resulting reaction force and tension force in the cable are the only forces present at joint A, they must add up to zero. Therefore, we can write:

$$T_{AB} = \sqrt{14.6^2 + 115.2^2} = 116.12\ kN \tag{8}$$

Similarly, we can determine the tension force in segment DE by calculating the vector sum of the reaction forces at joint E, as shown below.

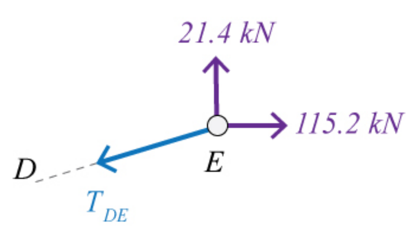


Figure 8: Free-body diagram of a cable joint

$$T_{DE} = \sqrt{21.4^2 + 115.2^2} = 117.17\ kN \tag{9}$$

To determine the tension force in segment BC, we need to examine the free-body diagram of the system to the left of joint C (see Figure 9).

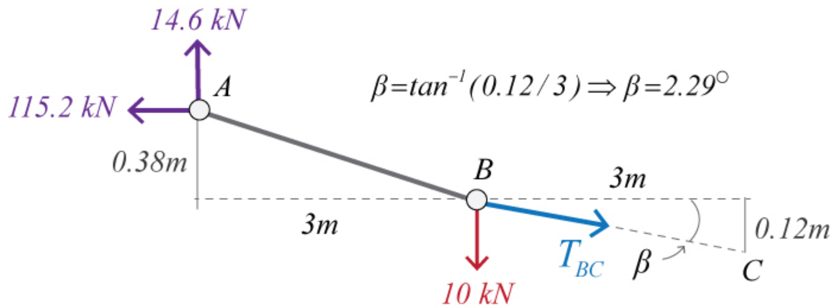


Figure 9: Free-body diagram of a cable subsystem consisting of one segment

Since the subsystem shown in the figure above must be in equilibrium, we can set the sum of the forces in the x-direction to zero. This produces the following equation.

$$\sum F_x = T_{BC} \cos(2.29) - 115.20 = 0 \qquad [10]$$

When Equation [10] is solved, it yields: $T_{BC} = 115.29\ kN$.

To determine the force in segment CD, let's consider the free-body diagram of the subsystem to the left of joint D (see Figure 10).

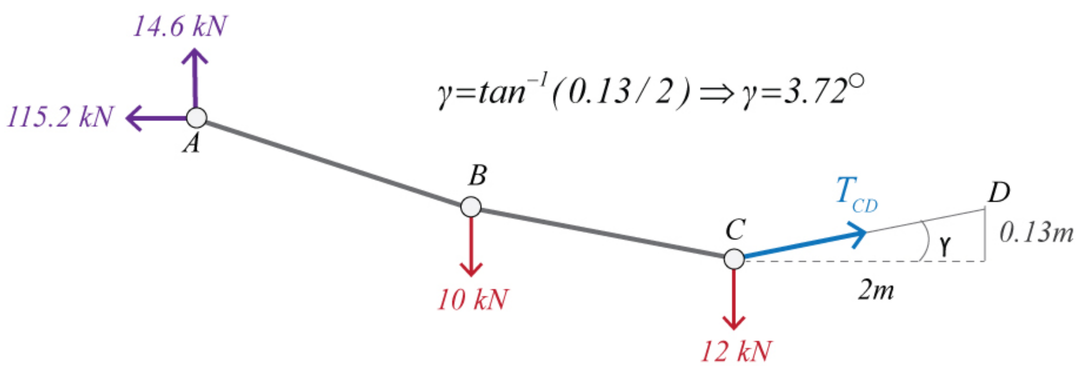


Figure 10: Free-body diagram of a cable subsystem consisting of two segments

Using the free-body diagram above, setting the sum of the forces in the x-direction to zero produces the following equilibrium equation.

$$\sum F_x = T_{CD} \cos(3.72) - 115.20 = 0 \qquad [11]$$

Solving the above equation for T_{CD}, we get: $T_{CD} = 115.44\ kN$. The results of this analysis are depicted in Figure 11.

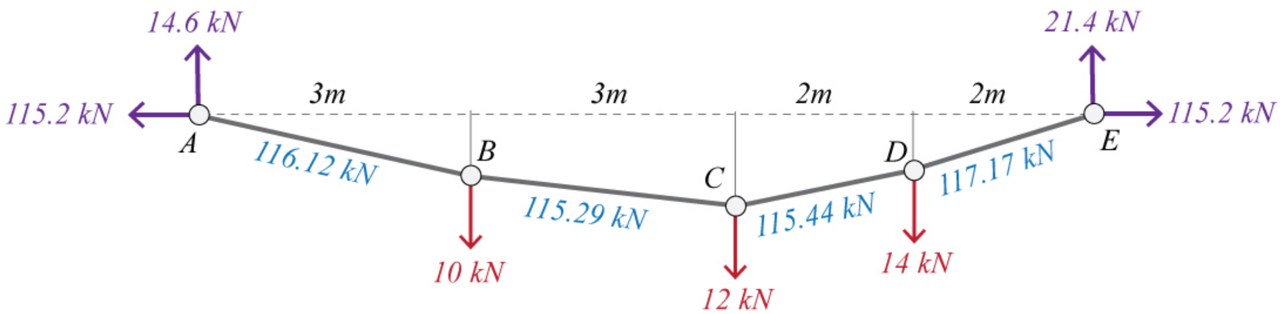


Figure 11: Internal and external forces associated with a cable system subjected to three loads

To summarize, given a cable system subjected to N joints and $N-2$ concentrated loads, positioned at heights h_2 through h_{n-1}, we can write three equilibrium equations for the system as a whole (see Equations [12] through [14]).

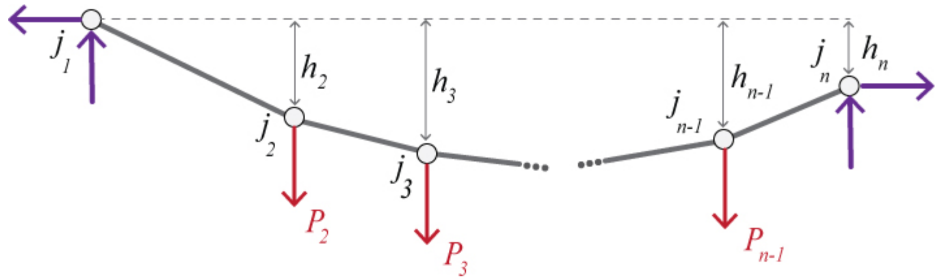


Figure 12: A cable system with N joints and N-2 concentrated loads

Furthermore, we can write one equilibrium equation for each interior joint (see Equation [15]). Therefore, since there are $N-2$ such joints, we end up with a total of $3+(N-2)$, or $N+1$ equations.

$$\sum F_x = 0 \qquad [12]$$

$$\sum F_y = 0 \qquad [13]$$

$$\sum M @ j_1 = 0 \qquad [14]$$

$$\sum M @ j_2 = \sum M @ j_3 =, \cdots, \sum M @ j_{n-1} = 0 \qquad [15]$$

The cable system has four support reactions and a total of $N-1$ unknown heights. Therefore, we could have up to $N+3$ unknowns. However, since the number of equations is 2 fewer than the maximum number of unknowns, we need to know two of the heights in order to be able to determine the rest of the unknowns.

Once all these unknowns are determined, we can use an equilibrium equation at each interior joint of the system to calculate the tension forces in the cable.

Let's use this process to solve the two exercise problems that were given in Lecture SA62.

Exercise 1: Analyze the cable system shown below.

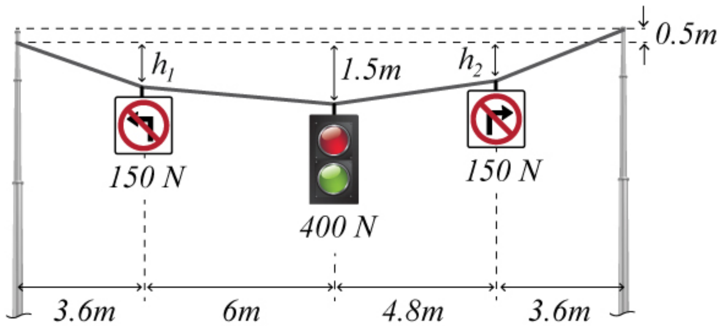


Figure 13: A cable system subjected to three concentrated loads

Since N (number of joints) equals 5, we can write $N+1$, or 6 equilibrium equations. As indicated in Figure 14, we have a total of six unknowns: four unknown support reactions and two unknown heights.

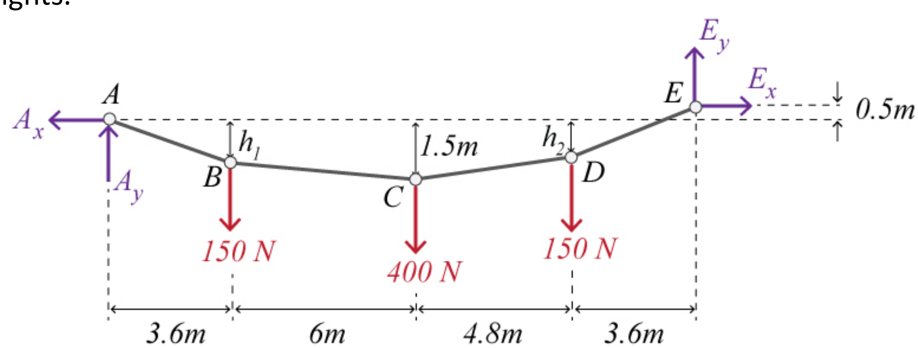


Figure 14: Free-body diagram of a cable system subjected to three concentrated loads

Using the free-body diagram shown above, we start by writing three equilibrium equations for the system as a whole (see Equations [16] through [18]).

$$\sum F_x = E_x - A_x = 0 \quad [16]$$

$$\sum F_y = A_y + E_y - 150 - 400 - 150 = 0 \quad [17]$$

$$\sum M @ A = 18E_y - 0.5E_x - 3.6(150) - 9.6(400) - 14.4(150) = 0 \quad [18]$$

If we cut segment BC and draw the free-body diagram of the left subsystem (see Figure 15), we can write Equation [19].

$$\sum M @ B = 3.6A_y - h_1 A_x = 0 \quad [19]$$

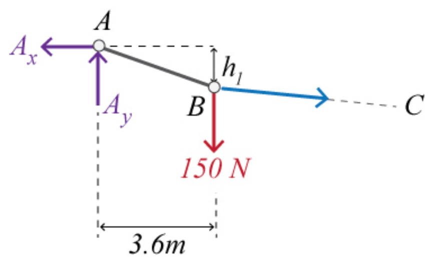


Figure 15: Free-body diagram of a cable subsystem consisting of a single segment

Furthermore, if we cut segment CD, we get the following free-body diagram, which yields Equation [20].

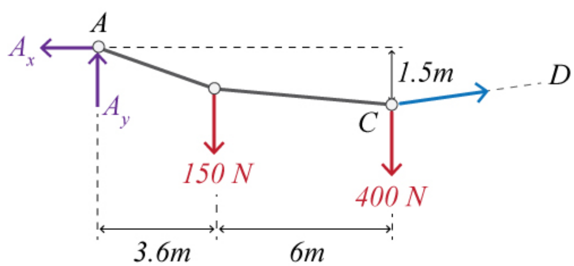


Figure 16: Free-body diagram of a cable subsystem consisting of two segments

$$\sum M @ C = 9.6 A_y - 1.5 A_x - 6(150) = 0 \quad [20]$$

Another equilibrium equation ([21]) can be written by cutting segment DE, as shown in Figure 17.

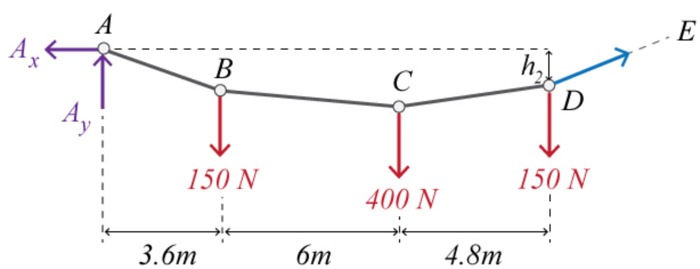


Figure 17: Free-body diagram of a cable subsystem consisting of three segments

$$\sum M @ D = 14.4 A_y - h_2 A_x - 10.8(150) - 4.8(400) = 0 \quad [21]$$

If we solve Equations [16] through [21] simultaneously, the following values result:
$A_x = 1320N$, $A_y = 300N$, $E_x = 1320N$, $E_y = 400N$, $h_1 = 0.818m$, $h_2 = 0.591m$.

Knowing the support reactions and the heights, we can now determine the tension force in each segment of the cable. To determine the force in segment AB, we can use the free-body diagram of joint A (see Figure 18).

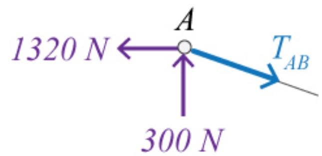


Figure 18: Free-body diagram of a left support point in a cable system

Since the sum of the forces at the joint must be zero, T_{AB} must be equal to the vector sum of the reaction forces at A. That is,

$$T_{AB} = \sqrt{1320^2 + 300^2} = 1354\ N \qquad [22]$$

Similarly, the tension force in segment DE (see Figure 19) must be equal to the vector sum of the reactions at E (see Equation [23]).

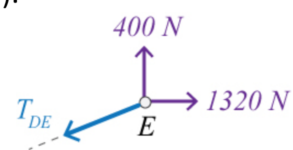


Figure 19: Free-body diagram of a right support point in a cable system

$$T_{DE} = \sqrt{1320^2 + 400^2} = 1379\ N \qquad [23]$$

To determine the force in segment BC, we can cut the segment and draw the free-body diagram of the subsystem to the left of point C, as shown in Figure 20.

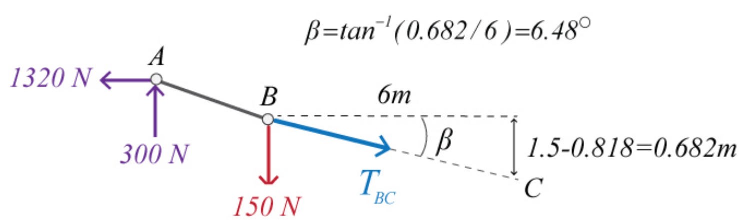


Figure 20: Free-body diagram of a cable subsystem consisting of a single cable segment

Since the conditions of equilibrium for the above subsystem must be maintained, we can write:

$$\sum F_x = T_{BC} \cos(6.48) - 1320 = 0 \qquad [24]$$

Solving Equation [24] for the unknown force, we get: $T_{BC} = 1328.5 N$.

Similarly, to determine the tension force in segment CD, we cut the segment and draw the free-body diagram of the subsystem to the left of point D, as shown in Figure 21.

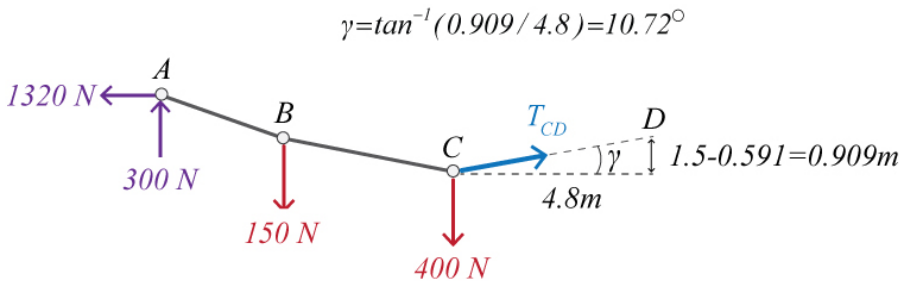


Figure 21: Free-body diagram of a cable subsystem consisting of two cable segments

Then, by setting the sum of the forces in the x-direction to zero, we can determine T_{CD}.

$$\sum F_x = T_{CD}\cos(10.72) - 1320 = 0 \Rightarrow T_{CD} = 1343.46\ N \quad [25]$$

The results of this analysis are depicted in Figure 22.

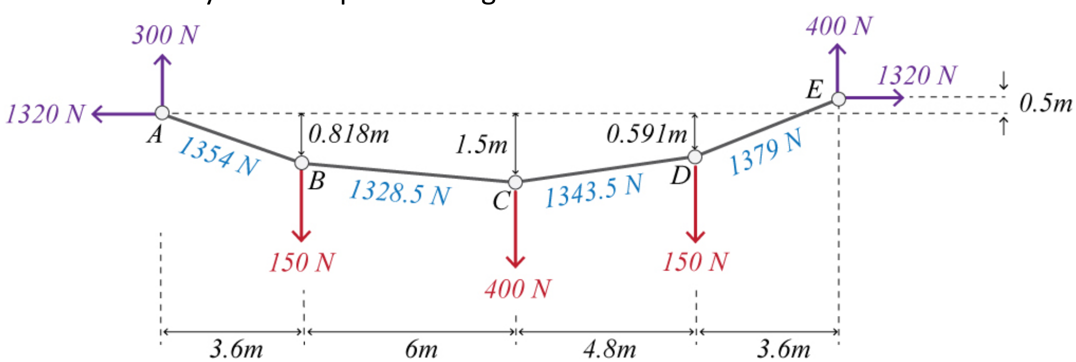


Figure 22: Support reactions and internal forces in an analyzed cable system

Exercise 2: Analyze the cable system and determine weight w for the cable system shown in Figure 23. The tension force in segment AB is known; it equals 130 N.

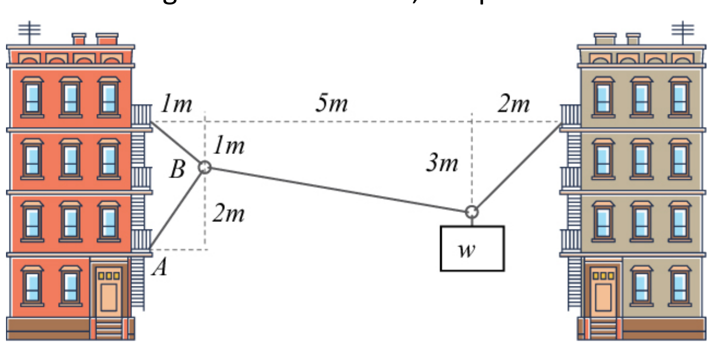


Figure 23: A cable system carrying a load with an unknown magnitude (w)

To solve the problem, we can start by drawing the free-body diagram of the system (see Figure 24).

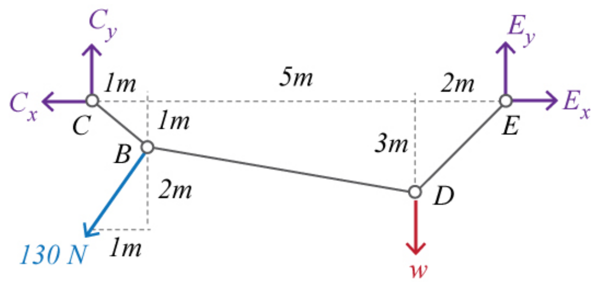


Figure 24: Free-body diagram of a cable system carrying a load with an unknown magnitude (w)

For convenience, let's replace the inclined force at joint B with its x and y components, as shown in Figure 25.

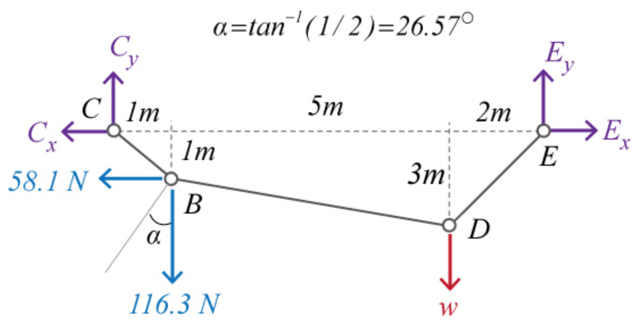


Figure 25: Free-body diagram of a cable system with forces in the x and y directions only

Five unknowns are present in the diagram above: four unknown support reactions and one unknown load (w). Since the cable system consists of four joints, we can write five equilibrium equations to determine the unknowns. Three of these equations are obtained by applying the conditions of equilibrium to the system as a whole. These equations are given below.

$$\sum F_x = E_x - C_x - 58.1 = 0 \quad [26]$$

$$\sum F_y = C_y + E_y - 116.3 - w = 0 \quad [27]$$

$$\sum M @ C = 8E_y - 58.1(1) - 116.3(1) - 6w = 0 \quad [28]$$

Another equation ([29]) can be obtained by cutting segment BD and summing the moments about point B (see Figure 26).

$$\sum M @ B = C_x - C_y = 0 \quad [29]$$

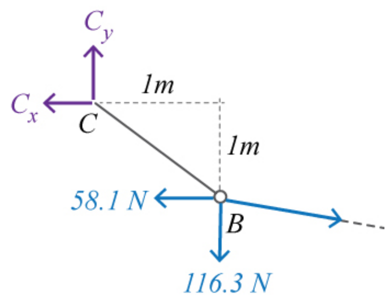


Figure 26: Free-body diagram of a cable subsystem consisting of a single segment

Lastly, a fifth equation ([30]) can be written by cutting segment DE and summing the moments about point D (see Figure 27).

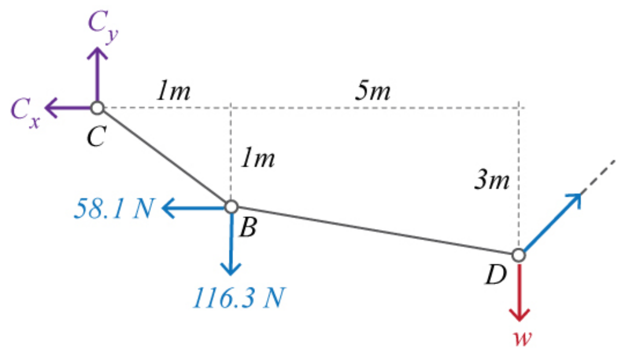


Figure 27: Free-body diagram of a cable subsystem consisting of a single segment

$$\sum M @ D = 6C_y - 3C_x - 58.1(2) - 116.3(5) = 0 \qquad [30]$$

Solving Equations [26] through [30] simultaneously, we get: $C_x = 232.57N$, $C_y = 232.57N$, $E_x = 290.67N$, $E_y = 436.00N$, $w = 552.27N$.

Knowing the support reactions and weight w, we can determine the tension force in each segment of the cable.

We start by drawing the free-body diagrams of joints C and E (see Figure 28). We can then calculate T_{CB} and T_{DE} using Equations [31] and [32] respectively.

Figure 28: Free-body diagrams of two support points in a cable system

$$T_{CB} = \sqrt{232.57^2 + 232.57^2} = 328.90\ N \quad [31]$$

$$T_{DE} = \sqrt{436^2 + 290.67^2} = 524\ N \quad [32]$$

To determine the tension force in segment BD, we cut the segment and draw the free-body diagram of the subsystem to the left of joint D (see Figure 29).

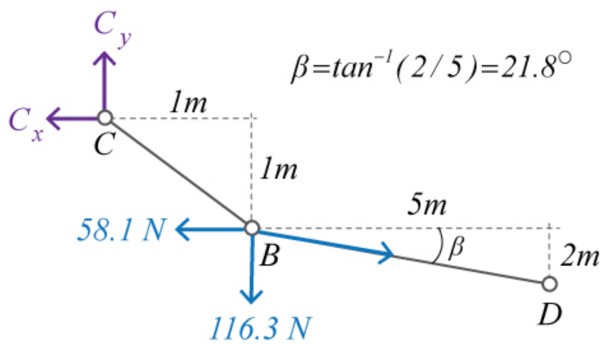


Figure 29: Free-body diagrams of a cable subsystem

Since the above subsystem must be in equilibrium, the sum of the forces in the x-direction must be zero. Therefore, we can write:

$$\sum F_x = T_{BD}\cos(21.80) - 232.57 - 58.1 = 0 \quad [33]$$

Solving the above equation for T_{BD}, we get: $T_{BD} = 313.06\ N$.

The following diagram shows the results of the analysis.

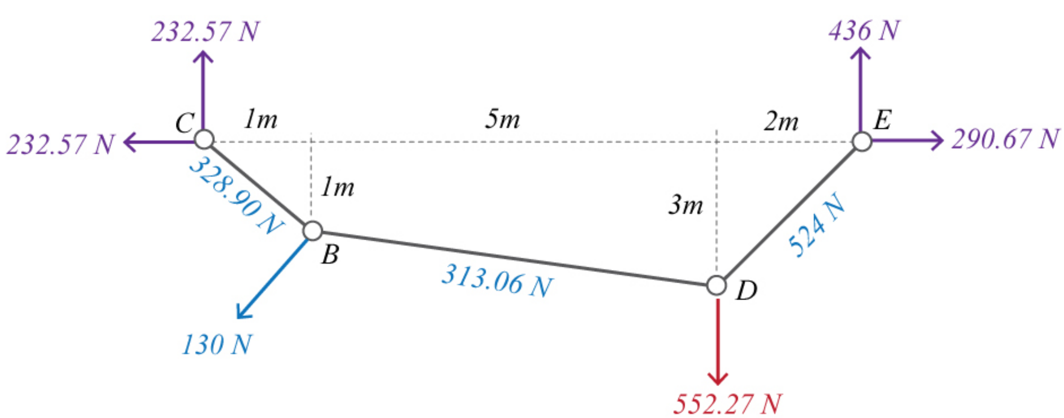


Figure 30: Support reactions and internal forces in an analyzed cable system

We will continue our discussion on the analysis of cables in the next lecture.