Engineering Mechanics: Statics
Lecture Series

ST11: Shear Diagram
Statically Determinate Beams

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Lab101.Space

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2019

Statics – ST11

Shear Diagram for Statically Determinate Beams

Prerequisite: ST10

In Lectures ST08 through ST10, we talked about shear force in beams, how to calculate it, and how to express it using algebraic equations. This lecture deals with the shear diagram, the graphical representation of the shear force in beams.

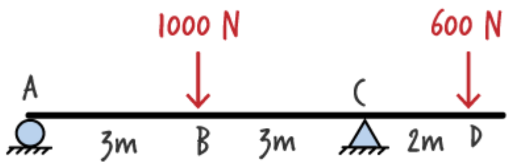

Figure 1: A simply supported beam

Consider the simply supported beam shown in Figure 1. Let us calculate the support reactions and write the shear equation for the beam. If you are not sure how to do this, please consult Lecture ST10.

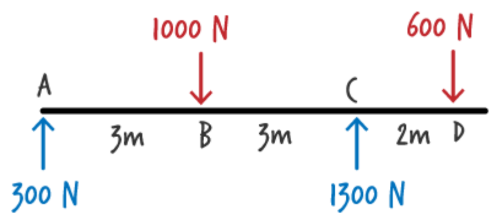

Figure 2: Free-body diagram for a simply supported beam

The support reactions for the beam are shown in Figure 2. The shear equation for the beam, derived from the above diagram, is as follows:

$$V(x)=\begin{cases} 300\text{ kN} & 0<x<3 \\ -700\text{ kN} & 3<x<6 \\ 600\text{ kN} & 6<x<8 \end{cases}$$

The graphical representation of this equation is the shear diagram for the beam. The above equation tells us that shear is constant in segments AB, BC, and CD. In AB, shear is 300 kN; in BC, shear remains -700 kN; and in segment CD, shear equals 600 kN.

The graphical representation of shear force in each segment is a straight line (see Figure 3).

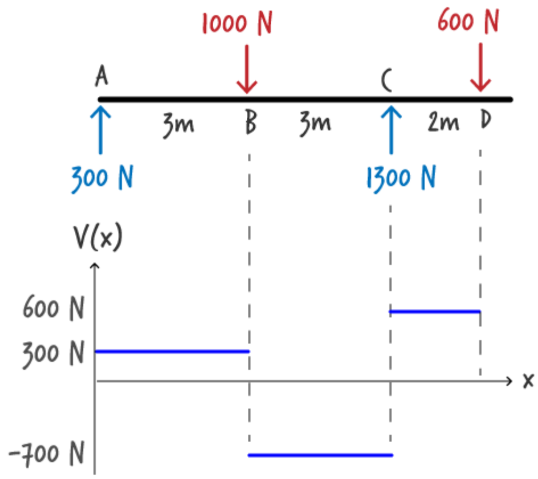

Figure 3: Graph of the shear equation

Then, the shear diagram can be drawn as shown below.

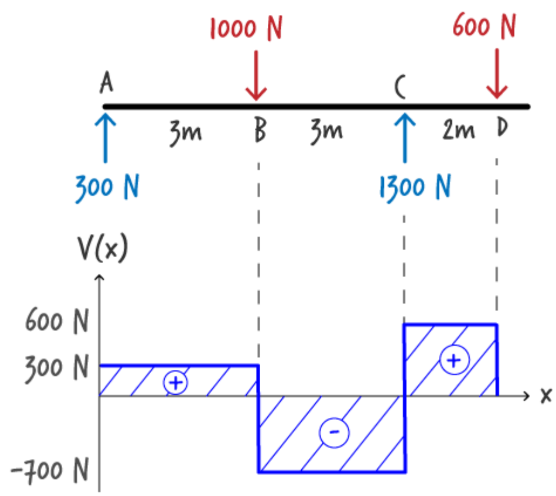

Figure 4: Shear diagram for the beam

As we can see, drawing the shear diagram from the underlying shear equation is rather straightforward and does not need much discussion. In this lecture, we are going to talk about the construction of the shear diagram directly from the free-body diagram of the beam, without using the shear equation.

Before we start, let's make an important observation. According to the shear equation given previously, shear is not defined at the points of application of the concentrated loads. The equation gives us the shear values within each segment, but not at its boundaries. Why is that? Why can't we define shear at the location of a concentrated load?

To see why, let's isolate Points A, B, C, and D, as shown in Figure 5.

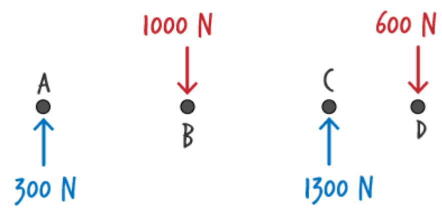

Figure 5: Four isolated points in the beam

At each point a concentrated force is present. These forces are external to the beam; they are acting on the beam and are not inside of it. In contrast, a shear force is internal to the beam. It does not show up in the free-body diagram unless we cut the beam.

Let's take a closer look at Point A. Since the point is dimensionless, we cannot break it up into smaller segments. Hence, for the static equilibrium to be maintained (for the sum of the forces in the y-direction to be zero) there must be an opposing force just to the right of A. Let's label that point A+ (see Figure 6).

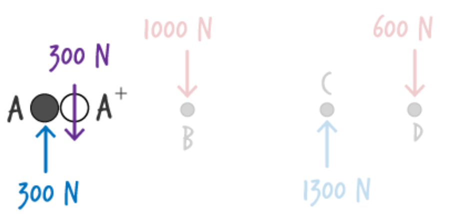

Figure 6: Point A in the state of equilibrium

So, a shear force must develop at A+ for Point A to remain in equilibrium. Since the magnitude of this force has to be 300 Newtons, we say that shear at A+ is 300 Newtons.

The same idea applies to Point B. Since the sum of the forces in the y-direction must be zero, a shear force on either side of B must develop. We refer to these points as B- and B+. In this case, shear at B- is 300 Newtons. At B+, we have a shear force of -700 Newtons (see Figure 7).

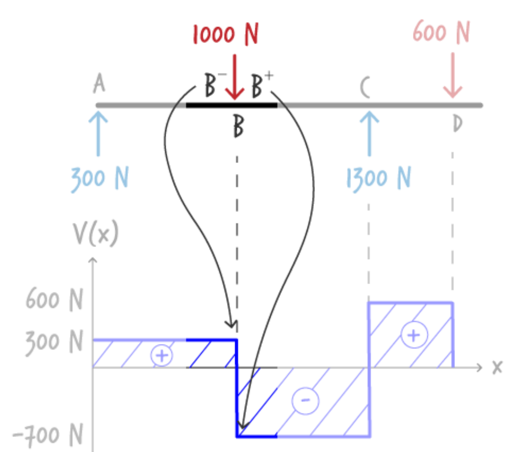

Figure 7: Shear at either side of Point B

Using the same line of reasoning, we can conclude that shear just to the left of C is -700 Newtons, but at C+ the force becomes 600 Newtons, and shear to the left of D is 600 Newtons, as depicted in Figure 8.

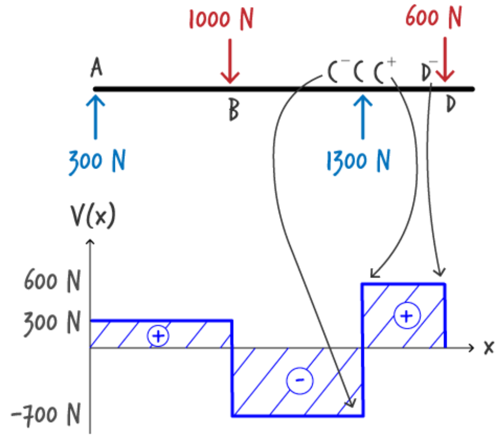

Figure 8: Shear near Points C and D

Now, let us devise a technique for constructing the shear diagram directly from the free-body diagram by closely examining the relationship between the two diagrams.

Starting from the left end and working toward the right end of the beam, we can correlate the diagrams in the following way:

There is an upward force of 300 Newtons at A. This corresponds to a jump in shear value, as the shear jumps from zero to 300 Newtons (see Figure 9). So, we can say that an upward external force on the beam causes an increase in shear value. The amount of increase is equal to the magnitude of the upward force.

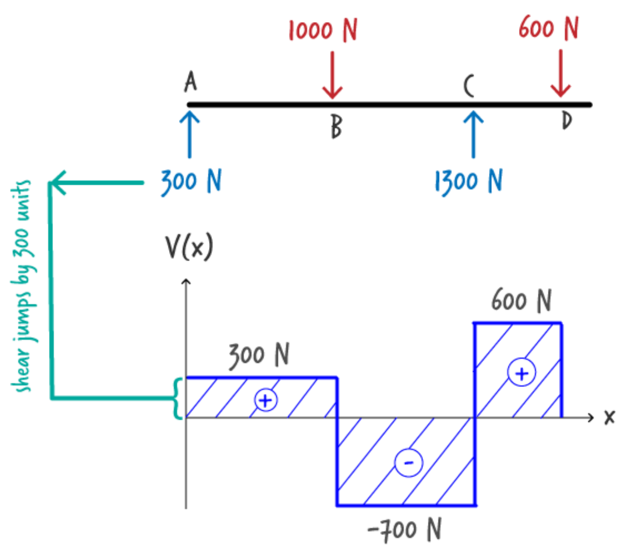

Figure 9: Jump in shear due to an upward force

The left beam segment (AB) is not subjected to any loads within it. That is, there is no load on the beam from A+ to B-. Therefore, no change in shear takes place. As you can see in Figure 9, shear in this segment of the beam has remained unchanged. The value of shear from A+ all the way to B- is 300 Newtons.

There is a downward force of 1000 Newtons at Point B, which causes a drop of 1000 Newtons in shear. Shear drops from positive 300 Newtons (at B-) to negative 700 Newtons at B+. Then, shear remains constant in BC, since the segment is not subjected to any external loads (see Figure 10).

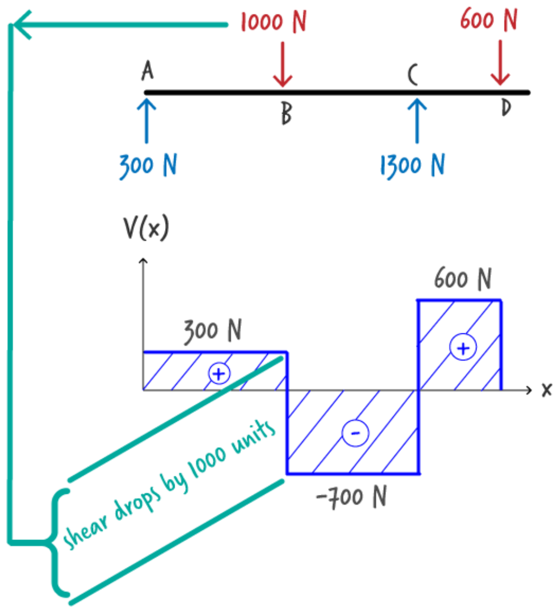

Figure 10: Drop in shear due to a downward force

Further, as can be seen in the figure above, the upward reaction force of 1300 Newtons at C causes an increase of 1300 units in shear value. It goes from negative 700 Newtons to positive 600 Newtons. Shear remains constant in segment CD before returning to zero at D.

This end value of zero represents shear at point D+. So, if we zoom in on Point D, we can see an upward shear force of 600 Newtons at D-, a downward applied force of 600 Newtons at D, and a zero shear force at D+ (see Figure 11).

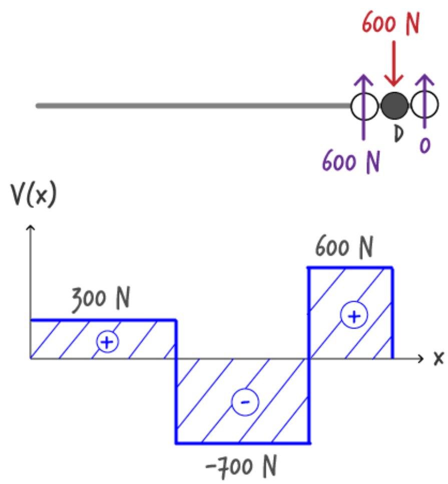

Figure 11: Shear force at Point D

Let's go through another example of how to draw the shear diagram directly from the free-body diagram for a beam. Consider the simply supported beam shown in Figure 12.

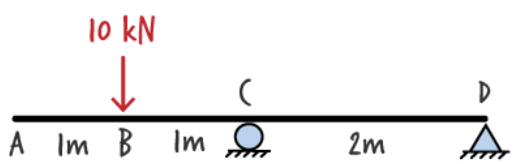

Figure 12: A simply supported beam with an overhang

To start, we draw the beam's free-body diagram, then write and solve the static equilibrium equations for the unknown support reactions. The calculated reaction forces are shown in Figure 13.

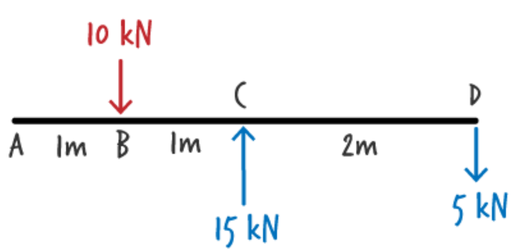

Figure 13: Free-body diagram for the beam with an overhang

Having the above diagram, we are now ready to draw the shear diagram for the beam. Let's start from end A and work our way toward end D.

Since there is no external force present at A, the starting value for shear is zero. Further, since segment AB is not subjected to any external loads, shear remains zero throughout the segment (see Figure 14).

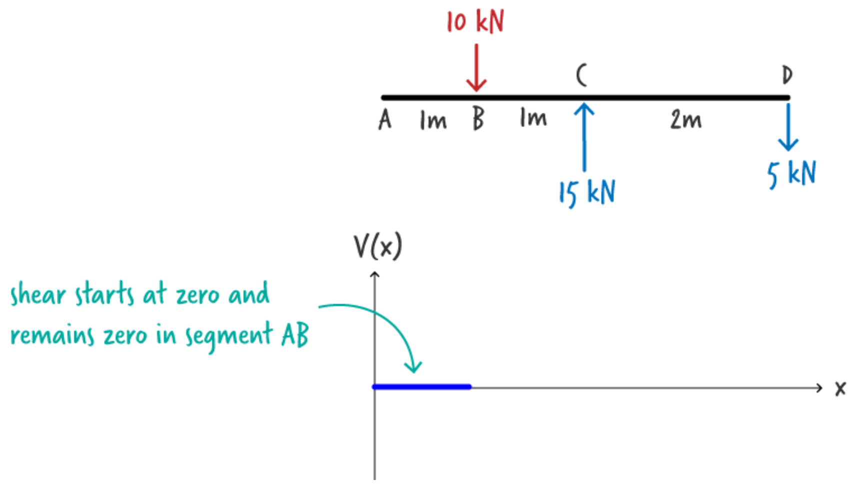

Figure 14: Shear diagram for segment AB of a beam

The downward force at B causes a drop in shear value; shear drops from zero to negative 10 kN. And since there is no external load placed on BC, shear remains constant throughout the segment, as depicted in Figure 15.

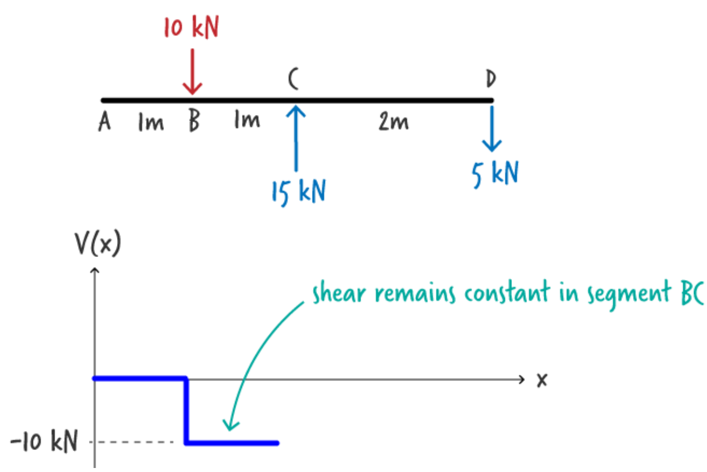

Figure 15: Shear diagram for segment BC of a beam

Since there is an upward force of 15 kN at C, there is going to be a jump of 15 units in shear value at that point; shear goes from negative 10 kN at the left side of C to positive 5 kN at the right side of C.

Shear remains constant in CD since the segment is not subjected to any external loads. At D, shear drops down to zero since there is a downward force of 5 kN present at that point. See Figure 16 for the graphical depiction of the shear force from C to D.

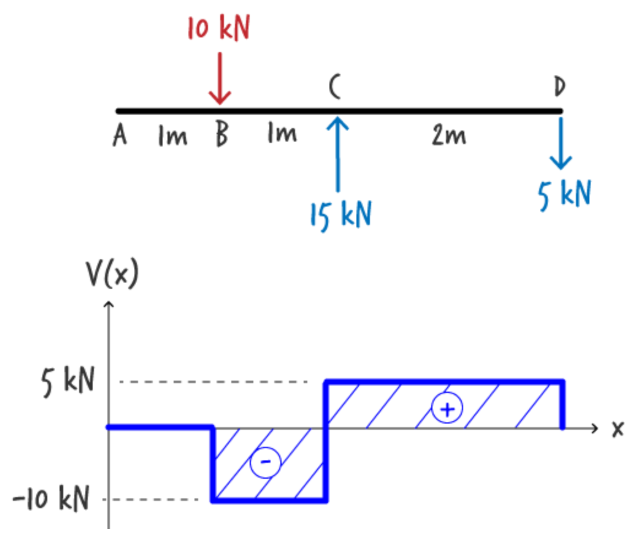

Figure 16: Shear diagram for a beam with an overhang

The shear diagram for the beam is shown in Figure 16.

Now let's look at a case in which the beam is subjected to a uniformly distributed load. The beam is shown in Figure 17.

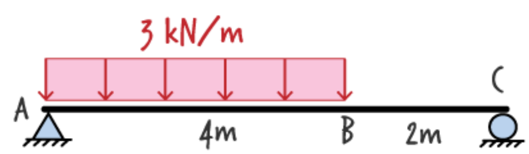

Figure 17: A beam subjected to a uniformly distributed load

The support reactions for the beam can easily be calculated by writing and solving the static equilibrium equations for the unknown forces. The calculated support reactions are shown in Figure 18.

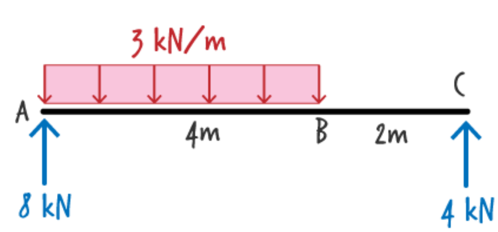

Figure 18: The free-body diagram of a beam subjected to a uniformly distributed load

Note how the distributed load divides the beam into two segments. The left segment is subjected to the load, but the right segment is load free.

To draw the shear diagram, we start from the left end of the beam. The upward force at A causes a positive shear force to develop at the left end of segment AB. The magnitude of the force is 8 kN (see Figure 19).

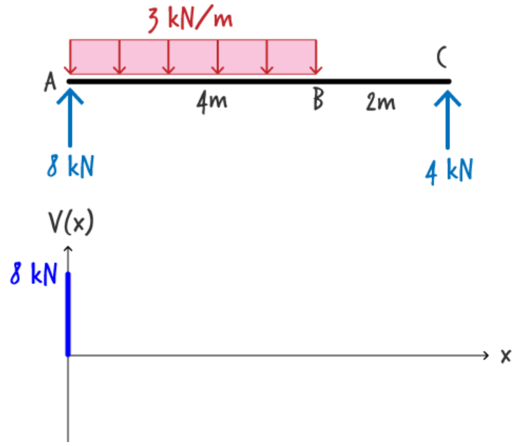

Figure 19: Shear at the left end of a simply supported beam

Since an external load is acting on AB, shear changes value within the segment. That is, shear at the right end of the segment has a different value than shear shown at the left end of the segment. The change in shear between the two end points is equal to the total area under the load diagram applied between A and B. This area is that of a rectangle with a height of 3 and a base of 4. So, the total area is 12. However, since the load is acting downward, we consider 12 to be a negative change in shear value. This means shear at the right end of the segment equals 8 minus 12, or negative 4 kN (see Figure 20).

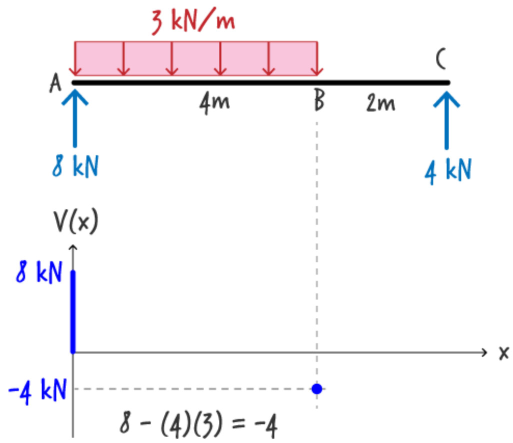

Figure 20: Shear differential between the ends of a beam segment

Since the distributed load has a constant magnitude, shear is going to change with a constant slope. That is, the diagram takes the shape of an inclined line, as shown in Figure 21.

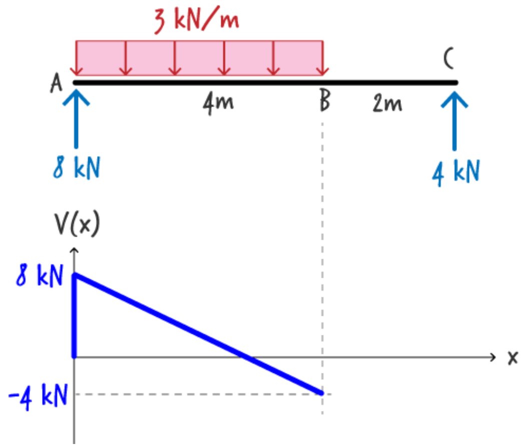

Figure 21: Shear diagram under a uniformly distributed load

It is important to keep in mind the rules for determining the shape of the shear diagram.

> *Rule 1: When there is no load applied to a beam segment, shear within the segment does not change value.*
>
> *Rule 2: When there is a constant (uniformly distributed) load applied to a beam segment, shear within the segment changes linearly.*
>
> *Rule 3: If the applied load is linear (i.e., a triangular load), then shear within the segment takes the shape of a quadratic equation.*

Shear within BC remains constant since the segment is not subjected to any loads. Finally, shear returns to zero since there is an upward force of 4 kN at C. The complete shear diagram for the beam is shown in Figure 22.

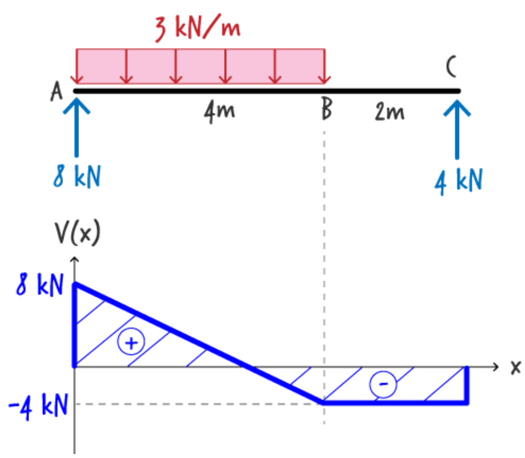

Figure 22: Shear diagram of a beam partially subjected to a uniformly distributed load

The following exercises should help solidify your understanding of the process described and illustrated in this lecture.

Draw the shear diagram for each beam shown below.