Structural Analysis

Lecture Series

SA03: Analysis of Beams having one or more Internal Hinges

This document is a written version of video lecture SA03, which can be found online at the web addresses listed below.

Educative Technologies, LLC

http://www.Lab101.Space
https://www.youtube.com/c/drstructure

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2019

Structural Analysis – SA03
Analysis of beams with one or more internal hinges

When designing structures it sometimes becomes necessary to connect two shorter structural members to form a longer one. Such a connection, also called a splice, can be classified as either a moment connection or a shear connection (see Figure 1).

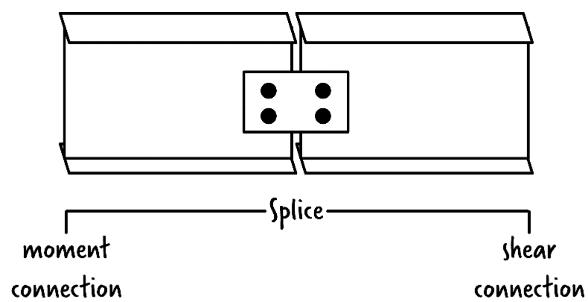

Figure 1: A splice of two connected beam members using a series of bolts

A moment splice rigidly connects the two members such that an internal bending moment as well as a shear force develops at the joint. See Figure 2 for an example of a moment splice.

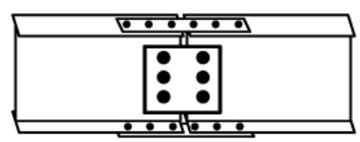

Figure 2: A moment splice with two members rigidly connected

In contrast, a shear splice allows relative rotation between the two segments and is less rigid than a moment splice, as depicted in Figure 3.

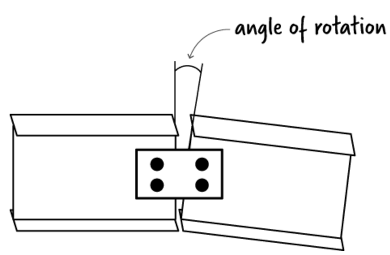

Figure 3: A shear splice allowing relative rotation at the joint

Consequently, no bending moment develops at the shear splice; the connection acts as an internal hinge transmitting a shear force only. Figure 4 shows the internal forces associated with each type of splice.

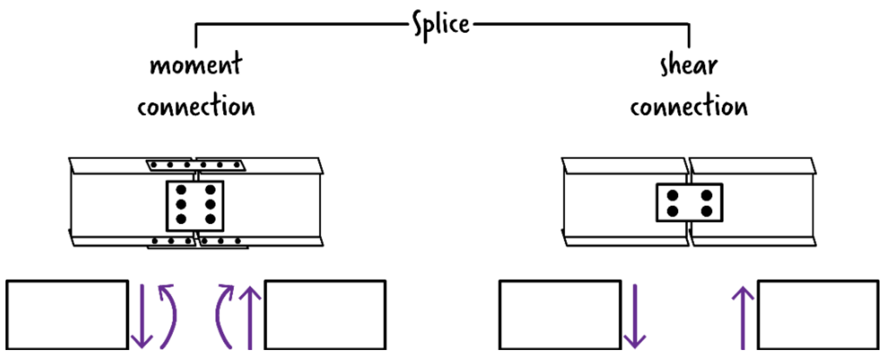

Figure 4: Internal forces in a moment connection and shear connection in beams

In this lecture, we're going to focus on the analysis of beams that embody one or more internal hinges. Consider the beam shown in Figure 5.

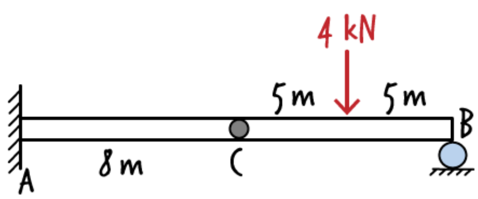

Figure 5: A beam with an internal hinge

The beam is fixed at the left end (point A) and rests on a roller support at the right end (point B). Furthermore, there is an internal hinge at point C. According to the free-body diagram of the beam (see Figure 6), there are four unknown support reactions: there are three at the fixed end and there is one unknown at the roller end.

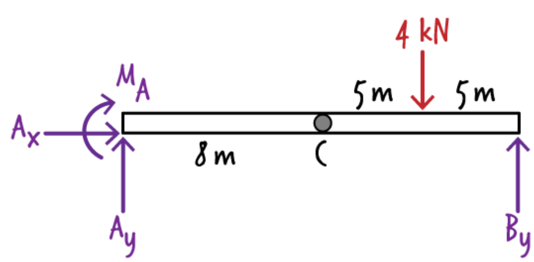

Figure 6: The free-body diagram of a beam with an internal hinge

We can write three equilibrium equations for the entire structure. However, these equations are not enough to determine all the unknowns because there are four unknowns and only three equations. Does this mean the structure is statically indeterminate?

This would have been the case, if there was no internal hinge in the beam. But, the presence of the hinge at point C makes the structure statically determinate. That is, we should be able to determine the support reactions solely using the static equilibrium equations. Here is how.

We separate the beam into two segments at the internal hinge (point C), as shown in Figure 7. Since no bending moment is present at the hinge, we get a total of six unknown forces for the left and right beam segments. Furthermore, since we can write three equilibrium equations for each segment, we get a total of six equations. Since the number of equations equals the number of unknowns, the unknowns can be found using the equations. Therefore, the beam is considered statically determinate.

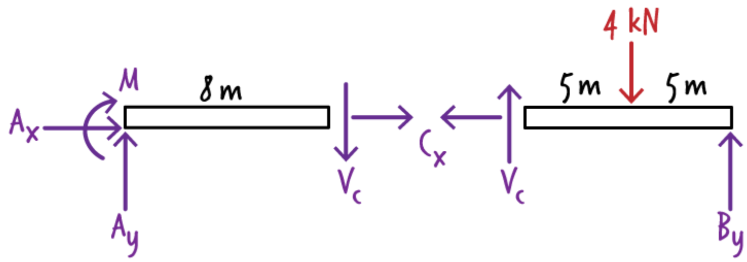

Figure 7: Internal forces in two beam segments connected at an internal hinge

Observe that the internal shear force at the hinge is drawn on each beam segment. On the left segment the force is shown to act downward. On the right segment the force is shown to act upward. These two arrows must always be drawn in opposing directions like we have shown here.

Also note that I have placed an internal axial force at point C. Although most beams do not carry such a force, for completeness purposes, we need to show the force on the free-body diagram. Similar to the internal shear force, the internal axial force at point C must appear in a pair acting in opposite directions, as depicted in the figure above.

The equilibrium equations for the left beam segment can be written as:

$$\sum F_x = 0 \Rightarrow A_x + C_x = 0 \quad [1]$$

$$\sum F_y = 0 \Rightarrow A_y - V_c = 0 \quad [2]$$

$$\sum M @ A = 0 \Rightarrow M + 8V_c = 0 \quad [3]$$

The equilibrium equations for the right beam segment are:

$$\sum F_x = 0 \Rightarrow -C_x = 0 \quad [4]$$

$$\sum F_y = 0 \Rightarrow V_c + B_y - 4 = 0 \quad [5]$$

$$\sum M @ C = 0 \Rightarrow 4(5) - 10B_y = 0 \quad [6]$$

Solving Equations [1] through [6] simultaneously for the unknowns, we get: $A_x = 0$, $A_y = 2\ kN$, $M = -16\ kN.m$, $B_y = 2\ kN$, $C_x = 0$, and $V_c = 2\ kN$. The support reactions for the beam are shown in Figure 8. Notice that the moment (M) at point A in Figure 8 is now drawn in the opposite direction than it was shown originally in Figure 7 when we first drew our internal forces. This is because we found out that the moment was actually a negative value after solving the equations for the unknowns, and therefore we changed the direction in which the moment was drawn from clockwise to counterclockwise.

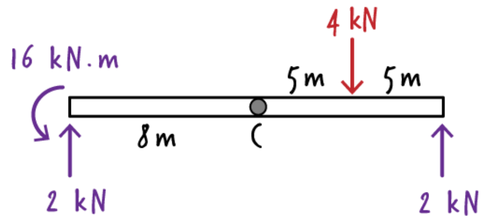

Figure 8: Support reactions for a beam with an internal hinge

What if the beam has multiple internal hinges? How do we analyze it? Figure 9 shows a beam with two internal hinges (one at point E and the other at point F).

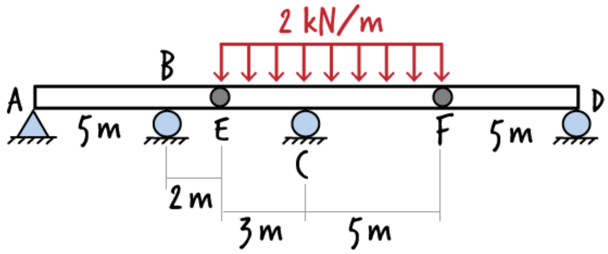

Figure 9: A beam with two internal hinges

The beam rests on four supports: a pin support (at point A) and three roller supports (one at point B, point C, and point D). Therefore, the number of support reactions is five; we have two reactions at the pin support at point A and one reaction at each of the three roller supports (see Figure 10).

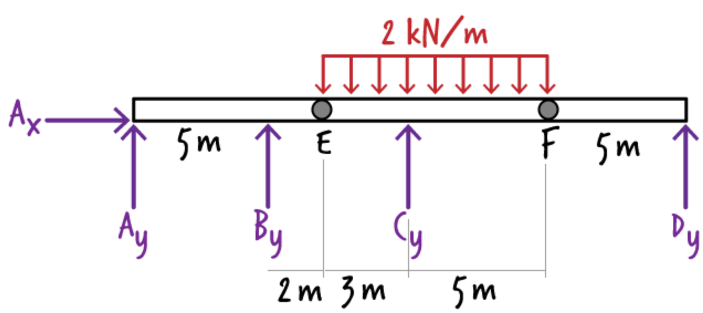

Figure 10: The free-body diagram of a beam with two internal hinges

Note that the two hinges divide the beam into three segments. If we cut the beam at the hinges, the following free-body diagrams result.

As shown in Figure 11, we are introducing a shear force and an axial force at each cut point. The internal forces at joints E and F plus the support reactions gives us a total of nine unknowns. Since we have three beam segments, and we can write three equilibrium equations for each segment, we get a total of nine equations. Given that the number of equations equals the number of unknowns, the beam is considered statically determinate.

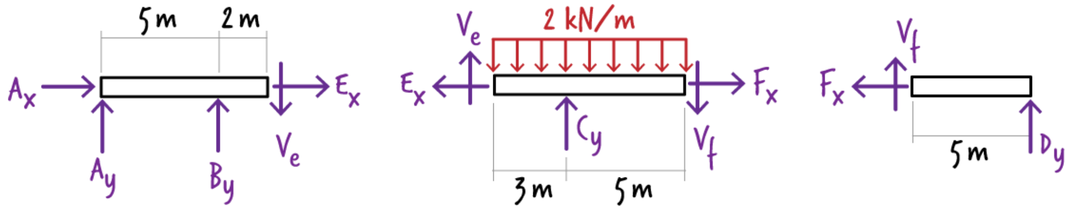

Figure 11: Free-body diagrams for three beam segments connected using two internal hinges

The equilibrium equations for segment AE are:

$$\sum F_x = 0 \Rightarrow A_x + E_x = 0 \quad [7]$$

$$\sum F_y = 0 \Rightarrow A_y + B_y - V_e = 0 \quad [8]$$

$$\sum M @ A = 0 \Rightarrow 7V_e - 5B_y = 0 \quad [9]$$

The equilibrium equations for segment EF are:

$$\sum F_x = 0 \Rightarrow F_x - E_x = 0 \quad [10]$$

$$\sum F_y = 0 \Rightarrow V_e + C_y - V_f - 2(8) = 0 \quad [11]$$

$$\sum M @ E = 0 \Rightarrow 2(8)(4) + 8V_f - 3C_y = 0 \quad [12]$$

And the equilibrium equations for segment FD are:

$$\sum F_x = 0 \Rightarrow -F_x = 0 \quad [13]$$

$$\sum F_y = 0 \Rightarrow V_f + D_y = 0 \quad [14]$$

$$\sum M @ F = 0 \Rightarrow -5D_y = 0 \quad [15]$$

Solving Equations [7] through [15] simultaneously, we get: $A_x = 0$, $A_y = 2.14\ kN$, $B_y = -7.47\ kN$, $E_x = 0$, $V_e = -5.33\ kN$, $C_y = 21.33\ kN$, $F_x = 0$, $V_f = 0$, and $D_y = 0$.

The results of the analysis are reflected in Figure 12.

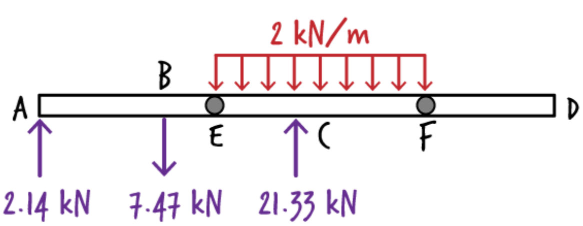

Figure 12: Support reactions for a beam with two internal hinges